The Capital Account and Aid Dependence

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ABSTRACT:
Closed capital accounts limit the scope for portfolio diversification. If further there is floor to consumption, at very low level of wealth uncertainty causes consumers to withdraw from investments. This paper studies the role of aid transfers aimed at fostering domestic capital accumulation in undoing this low-level development trap. Using mean-variance approach we derive optimal aid dependence and contrast investment policies in closed and open capital account regimes. It is shown that under free capital mobility: (*i*) the combination of investment risk and diminishing returns may reduce the proportion of aid funnelled to domestic investment as part of aid is invested abroad if not left idle; and (*ii*) if aid inflows are positively correlated with returns from foreign assets, consumers opt for less aid dependence because it is optimal to purchase foreign assets and insure themselves against risk.

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1. Introduction

Closed capital accounts limit the scope for portfolio diversification, exposing residents to undue levels of domestic risk. Cross-country studies find that uncertainty — either due to political instability, bad economic policies, or caused by exogenous shocks in vulnerable economies — reduces investment and growth. Further, poorly performing economies that find themselves unable to mobilise investment and become aid dependent are often those that impose distortions such as trade barriers and foreign-exchange controls in their dealings with the rest of the world.[1] This low-level development trap can be explained by the existence of a floor to consumption expenditures (Steger 2000). Good economic performance enables consumption to grow above its habitual level. Savings are released and consumers become less risk averse. In bad times, consumers seek to protect their living standards, reducing exposure to risk. In fact, empirical studies find a positive correlation between economic growth and savings, with the causation most likely to be running from the former to the later (Carrol, Overland, and Weil 2000).

The proposition that when living standards set a floor to current consumption, the proportion of wealth invested increases as consumers' budget constraints are relaxed is cornerstone to a number of development theories. Particularly, dual-economy theorists assume that investment is only undertaken from profit income. Because capitalists are rich they possess high marginal propensity to save, while labour income is hardly enough to meet workers' subsistence. Therefore, underdevelopment is a result of a virtually inexistent capitalist class facing a large pool of semi-employed agrarian labour force. For many, and beginning with the writings of Lewis (1954) and Rosenstein-Rodan (1961), this low-level development trap could be undone by aid transfers from capital-rich to capital-poor countries.

But a question has troubled economists ever since: will all aid injections be fully translated into investment in recipient economies? Soon sceptics argued, following an hypothesis suggested by Haavelmo (1963), that aid injections may actually produce consequences contrary to the intended objectives by reducing domestic savings. The discussion continued spilling to wider questions, including the one widely known these days: does aid promote growth?[2] However, whether there will be a one-for-one relationship between aid and investment is not the right question to ask. To the extent that both present and future consumption are normal goods, optimality conditions rule that both should expand as the budget constraint is relaxed. This result is known in the literature after the papers of Rahman (1967) and Griffin (1970).

This paper considers one aspect so far ignored in the debate — the degree of capital mobility between the recipient economy and the rest of the world — and makes two contributions. First, we establish the existence of optimal levels of aid dependence. This may sound controversial in so far as to the extent that aid inflows relax recipients' budget constraints more of them are preferable to less. However, it has been well documented that aid is volatile, and often more so than domestic sources of income in countries that depend on it most. This volatility is unwelcome to risk-averse consumers because it exacerbates the volatility of their income streams. Hence our result shows that aid volatility reduces the optimal level of dependence. The best aid inflows are those that are

[1] See Sachs and Warner (1995).

[2] See World Bank (1998).

counter-cyclical with the home economy to provide effective insurance to consumer-investors.

Second, we show that with capital mobility allowed, the combination of investment risk and diminishing returns in the recipient country reduces the proportion of aid funnelled to domestic investment. Part of aid will be invested abroad if not left idle. In order to achieve higher and stable consumption streams, investors seek those investment opportunities that promise high returns, but also diversify their portfolios to insure themselves against risk. As aid relaxes the budget constraints, the marginal unit of wealth must be used in order to satisfy these criteria. Only this way will the overall portfolios remain optimal.

These results are obtained in a model with habit persistence in consumption and risky investment opportunities. When the economy is closed, the only investment opportunity is on domestic perfectly-competitive firms that combine labour and capital in a neoclassical technology to produce a multi-purpose commodity that can either be consumed or used as a capital good. Production is subject to identically and independently distributed shocks. Because labour income is guaranteed, say, by contract, uncertainty ultimately impinges on returns to capital. With capital mobility allowed, residents can also trade on assets issued by firms in the rest of the world. Since the home economy is small, financial assets in the international markets are exogenously given.

Habit persistence has been used to solve a number of puzzles found under the traditional neoclassical model. In simulations exercises, Sundaresan (1989) showed that habit persistence solves the puzzle of ‘excess smoothness’ in consumption. The explanation for this ‘smoothness’ lies on the fact that while consumption resists falling below the current standards of living, it will rise only at a lower magnitude with income. An increase in current consumption raises utility given the standards of living, but also raises standards of living contributing to a decrease in future utilities. Low variability in consumption is attained at higher measures of risk aversion than implied by models with intertemporally separable preferences toward consumption. In fact, with adequate parameterisation, Constantinides (1990) showed that habit persistence in consumption resolves the equity premium puzzle.

Kraay and Ventura (2000) retained the assumption of intertemporally separable preferences toward consumption, but allowed a constant floor to consumption motivated by the need to meet subsistence requirements. They showed that even under perfect capital mobility the correlation between domestic saving and investment need not be zero, solving the Feldstein-Horioka puzzle. Diversification motive prompts domestic residents to invest part of their windfalls at home, not everything abroad as the traditional models suggest that a surplus in the current account of the balance of payments should follow. However, the existence of a constant floor to consumption makes the proportion of domestic risky capital an increasing function of wealth due to its effect on risk aversion.

The rest of the paper is organised as follows. Section 2 argues that intertemporally dependent preferences toward consumption are better placed than the traditional assumption of intertemporally separable preferences to explain cross-country differences in saving and risk-taking behaviour and, hence, the role of aid inflows. Section 3 puts forward the model and solves the consumer problem in both cases of closed and open capital account. Aid dependence is studied in section 4. Finally, section 5 summarises the main implications.

2. The Case for Intertemporally Dependent Preferences

Countries that have received aid transfers for prolonged periods of time are characterised by low or negative saving rates. Such long-term cross-country differences in saving rates cannot be explained by the neoclassical model of optimum growth, as proposed by Ramsey, Cass, and Koopmans. Because consumers are intertemporal optimisers, at each instant of time, consumption (c) is a constant proportion of wealth (W),

$$[1] \qquad c = \phi \cdot W,$$

the savings account records transitory variations to their lifetime wealth. In the long term, the net effect of these transitory variations should be zero. Only if we allow demographic factors such as in the life-cycle model, in which a positive saving rate is expected in economies with large cohorts of the young who are accumulating to provide for their old age. However, in cross-country comparison, this prediction does not stand the fact that poor countries have young population structures and yet they save less.

2.1 What Commands Time Preference?

The realisation that saving rates are higher in rich countries has led to the suggestions that the poor are impatient. But this statement is fraught with empirical difficulties. One cannot identify exactly what commands time preference: is it the discount rate or the intertemporal elasticity of substitution? To make this point clear, let us assume, as it has been the case in the literature, that preferences towards consumption are isoelastic:

$$[2] \qquad u(c) = \frac{c^{1-\theta} - 1}{1 - \theta}$$

Defining ξ as intertemporal elasticity of substitution in consumption, then [2] implies that ξ is constant: $\xi = -u'(c)/[cu''(c)] = 1/\theta$, which is why it is called the constant intertemporal elasticity of substitution in consumption. If financial wealth, a, earns a rate of return r, then the optimal growth path implies that consumption evolves according to the following Euler equation, also known as Keynes-Ramsey rule:

$$[3] \qquad \dot{c}/c = \xi(r - \delta)$$

where $\dot{c} = dc/dt$. The implication is that consumption grows as the rate of return on wealth rises ahead of the time preference rate (δ).

Using survey data on consumption expenditures in the United States of America, Lawrance (1991) estimated the Euler equation [3], conditioned by a set of socio-economic variables. The results showed that at a given rate of interest, food consumption expenditures grew with real labour income and with college education, but consumption growth rate was lower for non-white, poorer households. From this finding, Lawrance drew the conclusion that the rich and college educated were more patient and suggested a number of implications. First, time preference is culturally acquired. Second, because it is difficult to borrow against future labour incomes, impatient individuals preferred to stick to jobs paying low wages but with flat paths, foregoing education and training investments that would enable them to earn higher wages in future. Third, at the policy level, because poor households display high marginal propensity to consume, public transfers to them would hamper private savings and capital accumulation.

Lawrance's study assumed a constant intertemporal elasticity of substitution. Suppose that, instead of [2], consumers' preferences take the form:

$$[4] \qquad u(c) = \frac{(c-z)^{1-\theta} - 1}{1-\theta}$$

If we interpret z as the subsistence parameter (in which case $z > 0$), and since the intertemporal elasticity of substitution now takes the form:

$$\xi = \frac{1}{\theta}\left(1 - \frac{z}{c}\right)$$

it then follows that poor households, having their consumption level close to z will display lower elasticity of intertemporal substitution in consumption, hence not allowing variability in their consumption patterns, than rich households.

Using Indian panel data, Ogaki and Atkeson (1997) allowed the elasticity of intertemporal substitution in consumption to vary with the level of income. They found that although the mean growth rate in consumption was the same between rich and poor households, rich households had a more volatile consumption. Therefore, Lawrance's (1991) result could also imply that it is the elasticity of intertemporal substitution in consumption that varies with income, not the rate of time preference.

Both the Lawrance (1991) and Ogaki and Atkeson (1997) studies assumed complete markets, but the smooth pattern of poor households' consumption could be justified by the fact that the poor face borrowing constraints. However, incomplete markets have two opposing effects on consumption. The first is to induce higher risk aversion on consumers who are close to their borrowing constraints. The second is when households have actually hit their borrowing constraints. In this case, consumption growth will more strongly depend on current income, which makes consumption more volatile (Ogaki and Atkeson 1997).

2.2 Other Difficulties

In fact, the neoclassical growth model has come under attack for empirical failures other than the above mentioned. The prediction of consumption smoothing [1] has led to puzzling findings in the US economy. First, studies that specify the stochastic processes of consumption and income, enabling them to disentangle permanent components from the transitory, find that transitory income shocks have impact on consumption. Also, an empirical study by Hall and Mishkin (1982) found that although food consumption was more strongly correlated with permanent income, consumption responses to transitory income movement were 'vigorous'. In their cross-country study of household saving in developing countries, Schmidt-Hebbel, Webb, and Corsetti (1992) proxying permanent income by the time trend of sample income, found that only 0.3 of income deviations from its time trend were saved. This low coefficient casts doubt on to the permanent income hypothesis.

Finally, given the postulate of constant and positive discount rate, the Keynes-Ramsey rule given by the Euler equation [3] implies that consumption should only grow if real interest rate exceeds the subjective discount rate. However, as Deaton (1987) notes, in the post World War II period, consumption has been growing in the US, except in the year 1974, despite the fact that real interest rates were negative in the periods 1955-59, 1968-69, and 1971-80.

2.5 Conclusion

We take the view that differences in saving rates between rich and poor countries are explained by a combination of different growth performances and the existence of a floor to consumption. At any time, this minimum level of consumption is given by an exponential average of past consumption levels. Bad economic performance may require current consumption to fall, but consumers protect their living standards by reducing savings or, in the extreme conditions, deaccumulating wealth. Poverty traps may result: investment is necessary if economic conditions are to improve, but no further sacrifices can be made once consumption has fallen to the level of acquired living standards.

The first systematic treatment of the habit persistence hypothesis was by Duesemberry (1949), who suggested two types of habit persistence. The first is a 'demonstration effect' accruing through constant contact with goods of superior quality. This effect causes consumers of certain classes in the society to increase consumption in response to consumption patterns of those with whom they come in frequent contact. The second force is the very fact that the principal goal in the society is the achievement of higher standards of living. The desire to get superior goods provides a drive to higher expenditures, which may be stronger than that arising out of the needs, which are supposed to be satisfied by expenditure.

3. The Basic Model

This section outlines a model of optimal consumption and portfolio choices in a small open economy with intertemporally dependent preferences in consumption. Essentially it brings two works together. The first is that of Constantinides (1990) who models wealth dynamics when consumers display habit persistence in a closed economy. That paper assumed that consumers could only invest in constant-return-to-scale technologies, with zero flow of labour income. Instead, we assume an open economy and introduce labour income as well as diminishing returns to both capital and labour in the spirit of the work by Kraay and Ventura (2000) who proposed a portfolio-choice approach to study the dynamics of a small open economy. The section begins with a description of consumer preferences and choices available to him, and then studies two separate regimes: closed capital account and perfect capital mobility.

3.1 Consumer Preferences and Wealth Dynamics

The economy is inhabited by a constant number of consumers with identical preferences, captured by a variation of [4],

[5] $$u(c) = (c - z)^{1-\theta} / (1-\theta)$$

with two modifications. One being for mathematical convenience in so far as whereas [4] has the property that $\lim_{\theta \to 1} u(c) = \ln(c - z)$, [5] doesn't; however, while in deterministic settings the functional form [4] presents no problem, its use in stochastic optimal control problems makes the task of getting a closed-form solution to c extremely difficult. In fact we are not aware of any work of this kind that has used [4], whereas the use of [5] is common. The second modification is that z is no longer a constant parameter. Following Constantinides (1990) we allow the living standards to change according to consumers' economic conditions. Hence z is an exponentially-smoothed average of past consumption levels, $z(t) = e^{-\alpha t} z(0) + \beta \int_0^t e^{\alpha(s-t)} c(s) ds$, for any $s < t$ and constants $\alpha, \beta > 0$. Differentiating z with respect to time:

[6] $$dz = (\beta c - \alpha z)dt$$

The parameters α and β play opposing roles in habit formation. While α measures the rate of decay of importance of past consumption experience in habit formation –- the higher value of this parameter, the lower the weight $e^{-\alpha(s-t)}$ of consumption experienced in the most remote past, s, in the computation of the habit index z –- β measures the speed of updates in habits to the most recent consumption experiences. In the case of $\beta = 0$, but $\alpha > 0$, current consumption experiences play no role in habit formation, which decays exponentially at the rate α, since it is given by $e^{-\alpha t}z(0)$. If, instead $\alpha = 0$, but $\beta > 0$, past and present consumption experiences are weighted equally, but z may explode to infinity as time lapses. For a reasonable dynamics of this economy, if consumers update their habits after most recent consumption experiences, then they must have some memory loss of consumption experienced in the most distant past: $\alpha \geq \beta > 0$.

When $z(0) = \beta = 0$, we get the case of time-separable preferences toward consumption. If $\alpha = \beta = 0$, we get a case similar to equation [4]. In this case –- treated by Kraay and Ventura (2000) –- consumers' risk aversion is a decreasing function of wealth. Under habit persistence however, it is not enough for wealth to grow: it must grow enough to lift consumption above its habitual level. A positive intertemporal elasticity of substitution requires $\theta > 0$ and that consumption never falls below the living standards.

Consumers inherit some financial wealth, $a(0)>0$, and then decide whether to invest. Financial wealth is always held in liquid assets, so consumers do not risk being unable of fulfilling their optimal asset allocation strategies because of liquidity constraints. The decision to invest expresses their willingness to bear the risk associated with the undertaken investment opportunity, as each investment opportunity i is subject to random shocks, ε_i. These shocks are serially uncorrelated $\{E[\varepsilon_i(s)\cdot\varepsilon_i(t)] = 0$, for s and t standing for different points of time $(s \neq t)\}$, with zero mean $\{E[\varepsilon_i(t)] = 0\}$ and unit variance $\{E[\varepsilon_i(t)]^2 = 1\}$. Their arrival is described by the Wiener process, $dz_i = \varepsilon_i(t)(dt)^{1/2}$.

By not investing, consumers leave their assets on safe applications, earning a risk-free rate of return, r. Clearly, this alternative to investment is a theoretical construct that cannot be found in reality. A proxy often used in the finance literature are US Treasury bills, but they are also subject to risk of, for example, inflation. In fact they do display some volatility, although their standard deviation is smaller than that of other securities, such as equity. In any case, there is somewhere consumers store their wealth when they decide not to invest. That is what is meant by risk-free application of wealth in this paper.

In addition to financial wealth, consumers are endowed with an infinitely elastic labour force. It is often believed that labour income is less risky than financial income (Kraay and Ventura 2000). Hence we shall assume a deterministic wage rate y, because it is, say, specified in employment contracts.[3] The present value of the flows of labour income constitutes human wealth, $h(0) = \int_0^\infty e^{-rt}y(t)dt$, which in steady state implies $y = rh$.

Any chosen consumption and investment policy is admissible only if it does not violate consumer's intertemporal budget constraint: $W(0) \geq \int_0^\infty e^{-rt}c(t)dt$, where $W(0) = a(0) + h(0)$. Because current consumption must never fall below the living standards, this constraint implies that: $W(0) > \int_0^\infty e^{-(r+\alpha-\beta)}z(t)dt$, for $r + \alpha > \beta > 0$. Hence, at each point of time we must have $(r + \alpha - \beta)W - z > 0$. Contrary to the usual condition that when preferences are intertemporally independent consumers 'inherit' a positive stock of wealth, in our planning problem the present value of wealth should be net of lifetime living standards:

3 In Breeden (1979), labour income is stochastic because consumers supply stochastic labour units.

[7] $$W(0)-\frac{z(0)}{r+\alpha-\beta}>0.$$

One way to motivate the initial condition [7] is: Suppose that wealth annuity was equal to the living standards, then consumers would not save, but the dynamics of this economy requires saving. Further, because, as we shall see below, the measure of risk aversion varies with economic conditions, when wealth annuity can only meet living standards, consumers become highly risk averse and withdraw from investments. Such possibility should be avoided.

It is the choice of policies concerning applications of financial wealth and consumption that will guide the dynamics of W. Consumers want these policies to be optimal in the sense that they maximise the accumulations of discounted utilities. We turn to this problem next.

3.2 The Case of Strict Capital Controls

Suppose in the home economy the government has a policy of strict capital controls. Residents cannot invest abroad. If they decide to invest, consumers can only purchase shares issued by perfectly competitive firms residing in the home economy. Jointly, these firms produce an aggregate amount, Q, of output, if total amounts of capital, K, and labour, N (the total number of workers in the economy), are employed. Since the technology in use is neoclassical, we can express the instantaneous per capita value of output as:

$$E[Q/N] \equiv E[q] = (kf'(k) + y)dt$$

where $k = K/N$ and $f(k)$ satisfies the following properties: $f(0) = 0$, $f'(k)>0$, $f''(k)<0$, $\lim_{k\to 0}f'(k) = \infty$, and $\lim_{k\to\infty}f'(k) = 0$.

Given that σ_k is the standard deviation of returns to capital, then the realisations of per capita output are given by $q = (kf'(k) + y)dt + k\sigma_k dw_k$. In turn, financial wealth sums the amount invested, k, and that left in safe application, b: $a = k + b$. Taking these facts into account, the dynamics of financial wealth is captured by the following Brownian motion:

[8] $$dW = \{a[\omega f'(k) + r(1-\omega)] + y - c\}dt + a\omega\sigma_k dw_k$$

The planning problem facing the consumer, given initial stocks, $W(0)$ and $z(0)$, is to maximise the discounted flow of future utilities:

[9] $$V\left[W(0)-\frac{z(0)}{r+\alpha-\beta}\right]=\max_{c\omega} E_0\left[\int_0^\infty \frac{e^{-\delta t}}{1-\theta}(c-z)^{1-\theta}\,dt\right]$$

subject to [6], [7], and [8]. The solution to this problem can be expressed by a current-value function given by the following Hamilton-Jacobi-Bellman equation:[4]

[10] $$\delta V=\max_{c\omega}\left\{\frac{(c-z)^{1-\theta}}{1-\theta}+\frac{\partial V}{\partial z}(\beta c-\alpha z)+\frac{\partial V}{\partial W}\left[a\left(\omega f'(k)+r(1-\omega)\right)+y-c\right]+ +\frac{1}{2}\frac{\partial^2 V}{\partial W^2}(a\omega\sigma_k)^2\right\}$$

In the language of asset pricing theory, the left-hand side describes the required return from holding an asset. The right-hand side consists of the payout (in terms of utility)

[4] Details for the derivation in the Appendix.

from holding the asset plus the expected rate of capital gain over the next instant of time (Turnovsky 1997).

Differentiating [10] with respect to *c* and *ω*, respectively, and evaluating the result at maximum:

$$[11.a] \quad \frac{\partial V}{\partial W} = (c - z)^{-\theta} + \beta \frac{\partial V}{\partial z}$$

$$[11.b] \quad \omega = \frac{f'(k) - r}{\psi \sigma_k^2}$$

As shown and interpreted by earlier literature (Sundaresan 1989, Detemple and Zapatero 1991), consumption path is optimal if it satisfies the condition that the shadow value of wealth must equal the marginal utility to current consumption net of the loss of future utilities due to the induced consumption habits [11.*a*]. Defining $\psi \equiv -\left(a \cdot \frac{\partial^2 V}{\partial W^2} \Big/ \frac{\partial V}{\partial W} \right)$ as the measure of risk aversion, that we shall assume remains constant, [11.*b*] says that the amount of wealth invested increases with excess returns, but it decreases with return volatility, the higher is risk aversion. From [11.*b*] we can also derive the impacts of wealth and capital-stock changes on investment decision and on capital stock itself:

$$\frac{\partial \omega}{\partial a} = 0; \quad \frac{\partial k}{\partial a} = \omega \left[\frac{\psi \sigma_k^2}{\psi \sigma_k^2 - a f''(k)} \right]; \quad \frac{\partial \omega}{\partial k} = \frac{f''(k)}{\psi \sigma_k^2} < 0$$

That is, investment decision is unaffected by absolute changes in financial wealth. However, as Kraay and Ventura (2000) showed, the impact of changes in financial wealth on capital stock depends on the combination of investment risk and diminishing returns. Suppose investment risk is low ($\sigma_k \to 0$), then shocks on financial wealth will not affect capital stock. Therefore, investment risk is necessary for changes in financial wealth to cause changes in domestic capital; however, diminishing returns reduce the degree at which capital responds to changes in financial wealth. To illustrate this, in the case of linear technologies [$f'(k) = 0$] of each unit of additional wealth, a proportion *ω* is invested, but in the case of diminishing returns, the proportion of additional wealth going to capital is lower. Decreasing returns also lead to a decrease in the proportion of wealth invested: an economy facing diminishing returns but cannot export its capital abroad may have to accumulate more idle resources.

As remarked earlier, the optimal consumption rule is provided by [11.*a*]. However a closed-form solution cannot be found unless we know the functional form of the value function $V(\bullet)$. We try the following solution:

$$[12] \qquad V = \frac{A}{1-\theta} \left(W - \frac{z}{r + \alpha - \beta} \right)^{1-\theta}$$

where *A* is a positive constant to be determined. Taking the necessary derivatives of [12] and substituting them into [11.*a*] and then substituting this result back into the optimal current value function [10], we get:

$$[13] \qquad c = z + \phi \left(W - \frac{z}{r + \alpha - \beta} \right)$$

where *ϕ* (derived in the Appendix) is the marginal propensity to consume of the excess of lifetime wealth to lifetime living standards. Hence, consumption may rise above living standards if wealth annuity surpasses living standards. Otherwise, consumption remains at the level *z*. As a result, under habit persistence consumption will exhibit low variability. Instead, wealth becomes highly volatile as it absorbs the largest proportion of shocks to allow consumption smoothness (Sundaresan 1989).

An important feature of [12], solving the consumption function, is that it implies that the measure of risk aversion is $\psi = \frac{(r+\alpha-\beta)\theta a}{(r+\alpha-\beta)W - z}$; hence, as wealth annuity approaches the level of living standards, consumers become highly risk averse. At that point, they withdraw form any investment decision (see [11.*b*]) preferring to leave their assets idle even if that means foregoing a potentially higher mean income. In order to ensure risk taking by these consumers, it is necessary that wealth annuity is always above their living standards, which is why we assumed the constraint [7].

3.3 The Case of Perfect Capital Mobility

When the capital account is perfectly liberalised, in addition to investment opportunities available at home, consumers can also invest in a composite index of shares issued by firms residing in the rest of the world. It is assumed that these firms also operate in a perfectly competitive environment. Because the home economy is small, trading by residents cannot affect returns to scale in the capital invested in the rest of the world. Hence, it is as if they were facing constant marginal returns, with mean r^*. If they invest an amount k^* of capital in the rest of the world, their flow of income will be: $r^*k^*dt + k^*\sigma_{k^*}dw_{k^*}$, where σ_{k^*} is the variance of the rate of return and dw_{k^*}, the Wiener process describing such returns.

Now, per capita stock of financial wealth is: $a = k + k^* + b$. If we further define $\omega^* = k^*/a$, we can express wealth dynamics as:

$$[8'] \quad dW = \{a[\omega f'(k) + r^*\omega^* + r(1 - \omega^* - \omega)] + y - c\}dt + a[\omega\sigma_k dw_k + \omega^*\sigma_{k^*}dw_{k^*}]$$

Maximising [9] subject to [6], [7] and [8'] we get the following rules for optimal portfolio:[5]

$$[14.a] \quad \omega = \frac{1}{(1-\rho_{k^*k}^2)\psi}\left[\frac{f'(k)-r}{\sigma_k^2} - \frac{\rho_{k^*k}}{\sigma_k}\left(\frac{r^*-r}{\sigma_{k^*}}\right)\right]$$

$$[14.b] \quad \omega^* = \frac{1}{(1-\rho_{k^*k}^2)\psi}\left[\frac{r^*-r}{\sigma_{k^*}^2} - \frac{\rho_{k^*k}}{\sigma_{k^*}}\left(\frac{f'(k)-r}{\sigma_k}\right)\right]$$

for $|\rho_{k^*k}| < 1$. The possibility of investing abroad increases the holding of both domestic and foreign capital if correlation between their returns is negative. The force in action here is risk aversion driving consumers to diversify wealth portfolios. A negative correlation between returns from wealth invested in the home economy and that invested abroad provides consumers with insurance. On average, one of these alternatives does well when the other performs badly. When portfolios are diversified this way, overall wealth volatility is reduced. However, with positive correlation between returns from both investment opportunities, investors decide to put most of their wealth based on relative volatilities. For example, facing investment risk at home while returns from foreign assets are almost certain, investors would rather put most of their wealth abroad.

From both [14.*a*] and [14.*b*] we can derive the effects of changes in wealth on both domestic capital and capital invested abroad, and of changes in domestic capital stock o wealth allocation:

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[5] Since the first-order condition for optimal consumption path is unaffected by the number of investment opportunities, reproducing it here would be repeating [11.a].

$$\frac{\partial k}{\partial a} = \omega\left[\frac{\left(1-\rho_{k^*k}^2\right)\psi\sigma_k^2}{\left(1-\rho_{k^*k}^2\right)\psi\sigma_k^2 - af''(k)}\right]; \qquad \frac{\partial \omega}{\partial k} = \frac{f''(k)}{\psi\left(1-\rho_{k^*k}^2\right)\sigma_k^2} < 0;$$

$$\frac{\partial k^*}{\partial a} = \omega^*; \qquad \frac{\partial \omega^*}{\partial k} = -\frac{\rho_{k^*k} f''(k)}{\psi\left(1-\rho_{k^*k}^2\right)\sigma_{k^*}\sigma_k}$$

Other things held constant, as financial wealth increases, diminishing returns cause domestic capital to increase at a lower rate than its share on optimal portfolio, unlike foreign assets. In fact, the proporton of domestic capital on wealth portfolios decreases as capital stock grows. The proportion of foreign assets will only increase if returns from both investment opportunities are correlated. Since the rate of return at home is declining, investors seek bettwer opportunities abroad. However, if these returns are negatively correlated, in which case the diversification motive is strong, the proportion of foreign assets declines too.

Kraay and Ventura (2000) asked the following question: suppose the economy receives a windfall in wealth, will it run a surplus in the current account of the balance of payments? This is an important question too in our analysis because an answer to it will enable us to assess the impact of aid inflows on the external accounts. The foregoing discussion suggests that it all depends on the value of ω^*. When $\omega^* > 0$, as we have been assuming, investors put some of the windfall abroad in order to keep their portfolios optimal. The resulting deficit in the capital account is mirrored by a surplus in the current account. However, when residents have accumulated liabilities the reverse should follow. Hence the Kraay and Ventura's rule for the evaluation of the impacts of wealth shocks on the current account: it is the saving generated by the shock multiplied by the share of foreign assets in domestic wealth.

Continuing to interpret the results, as capital stock increases, the proportion of domestic capital in the wealth portfolios decreases as the proportion of wealth invested abroad increases. The benefit of free capital mobility is a reduction in the proportion of wealth held idle:

$$\frac{\partial(\omega^* + \omega)}{\partial k} = \frac{\left(\frac{\sigma_{k^*}}{\sigma_k} - \rho_{k^*k}\right) f''(k)}{\psi\left(1-\rho_{k^*k}^2\right)\sigma_{k^*}\sigma_k} < 0$$

if $\sigma_{k^*} > \rho_{k^*k}\sigma_k$ and $|\rho_{k^*k}| < 1$. Comparing this result with $\partial\omega/\partial k$ in the case of closed capital account, we see that although the total amount invested declines as domestic capital stock increases, the decline is lower than in the case of a closed economy.

3.4 Conclusion

Habit persistence makes risk aversion endogenous to consumer's economic conditions. Risk aversion declines in booms and rises in recessions. In an uncertain environment, shocks of unpredicted magnitude may materialise to reduce income from living standards. The model presented in this section predicts that consumers will withdraw from investment, preferring to remain solely labourers because the wage rate is certain. However, if production is to take place, labour must be combined with capital. Hence, aid inflows have a potentially important role in undoing this poverty trap.

A question has troubled economists interested in financing development for many years: suppose a unit of aid is transferred to poorer economies, will it translate into a unit of investment? From the foregoing analysis, we may conclude that to the extent that aid relaxes budget constraints, part of it will be consumed. If capital mobility is perfect, part of the amount devoted to investment will be invested abroad. Therefore, we cannot expect a one-for-one-relationship between aid and investment. In the next section, we turn to aid dependence.

4. Optimal Aid Dependence

Since the writings of Lewis (1954) and Rosenstein-Rodan (1961), aid injections have been advocated as means for lifting poor countries from underdevelopment. Early development theorists viewed underdevelopment as a process caused by the shortage of capital necessary to match the abundant labour force. From the very beginning, the role of aid on economic development caused controversy. Advocates saw foreign aid – which would be fully translated into capital accumulation – playing a complementary role to domestic savings. Critics, following Haavelmo (1963) have argued that aid injections were deleterious to development as they substituted for domestic savings. It has been long demonstrated that to the extent that both present and future consumption are normal goods, both should expand as aid injections lift the budget constraint.[6]

Rather, this section tackles a different question. Suppose consumers expect to receive an accumulated stock of transfers, $\zeta(t)$, over a period $[0,t]$, how would that affect their risk-taking behaviour? Empirical studies have established some well-known facts: aid is volatile, and even more volatile than domestic variables like fiscal revenues; aid volatility increases with aid dependency defined as a share of aid on, say, national income; aid inflows are pro-cyclical to economic performance in the recipient economy (Bulír and Hamann 2003). The question, then is what is the potential effects of these facts about aid on asset allocation and the implied excess returns?

We assume that consumers know that x is the instantaneous mean of aid inflows, while σ_x is the standard deviation of the instantaneous growth rate of these aid inflows. Therefore the disbursement process takes the following Brownian motion:

$$d\zeta(t) = x(t) + x(t)\sigma_x dw_x$$

Because aid relaxes the budget constraint to $W = a + h + \zeta$, more of it is preferred to less. It enables consumers to move to indifference curves as high that could otherwise be unachievable. However, volatility makes aid dependence a non-trivial matter. Consumers dislike risk. Then we shall propose the existence of an optimal level of aid dependence, x/a, for each level of volatility, σ_x. Of course, the feasibility of such a level hinges on the assumptions that consumers can shop around for donors and choose those with conditions that exactly match their preferences and that there are no indivisibilities in aid.[7]

4.1 Aid Dependence with Closed Capital Account

With capital mobility disallowed, wealth dynamics becomes:

$$[8''] \qquad dW = \{a[\omega f'(k) + r(1-\omega) + x/a] + w - c\}dt + a[\omega\sigma_k dw_k + (x/a)\sigma_x dw_x]$$

and, in turn, the maximisation of [9] subject to [6], [7], and [8''] yields the following rules for optimal portfolio and aid dependence:

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6 See Rahman (1967) and Griffin (1970).

7 Even if these assumptions are unrealistic, the result is still important because it provides the rules that donors interested on recipients' welfare should follow.

$$[15.a] \quad \omega = \frac{1}{\left(1-\rho_{kx}^2\right)\psi}\left[\frac{f'(k)-r}{\sigma_k^2} - \frac{\rho_{kx}}{\sigma_k}\cdot\frac{1}{\sigma_x}\right]$$

$$[15.b] \quad \frac{x}{a} = \frac{1}{\left(1-\rho_{kx}^2\right)\psi}\left[\frac{1}{\sigma_x^2} - \frac{\rho_{kx}}{\sigma_x}\left(\frac{f'(k)-r}{\sigma_k}\right)\right]$$

Again, we shall impose the requirement that $|\rho_{kx}| < 1$; otherwise, the optimal levels of ω and x/a become indeterminate.

Optimal aid dependence is decreasing with aid volatility. Notice that with zero variance, optimal aid dependency becomes infinite. This is the case in which more aid is preferred to less. When investment risk is extremely high ($\sigma_k \to \infty$), such as in the case of war or other calamities that make future prospects of the economy exceedingly difficult to discern, parameters of the home economy become irrelevant in determining optimal aid dependence. Under normal conditions, when σ_k is finite, the impact of the home economy in determining optimal aid dependence depends on the correlation between aid inflows and returns to domestic capital. In the ideal world, aid inflows should be counter-cyclical ($-1 < \rho_{kx} < 0$), because donors help economies in distress and reduce their support as conditions improve. In such a case, optimal aid dependence increases with growth prospects –- the excess of marginal product of capital to the risk-free interest rate –- given investment risk. Economies with bright prospects have higher mean incomes in the long run. Since counter-cyclical aid plays an insurance role, it should be higher to ensure those mean incomes. On the contrary, if aid is pro-cyclical ($0 < \rho_{kx} < 1$), optimal aid dependence declines with growth prospects. Because volatile aid does not provide them with insurance, economies with bright prospect would do with little of it.

The level of aid does not matter for optimal portfolio selection, but its volatility and correlation with returns to domestic capital. Counter-cyclical aid increases the proportion of capital on wealth portfolios, but its volatility reduces the amount of wealth invested. Only in the case of pro-cyclical aid does aid volatility increase the proportion of wealth invested, because in that case consumers prefer to rely less on aid.

The impacts of shocks to financial wealth and changes in domestic capital are as follows:

$$\frac{\partial x}{\partial a} = \frac{x}{a}; \qquad \frac{\partial(x/a)}{\partial k} = -\frac{\rho_{kx} f''(k)}{\psi\left(1-\rho_{kx}^2\right)\sigma_k\sigma_x};$$

$$\frac{\partial k}{\partial a} = \omega\left[\frac{\left(1-\rho_{kx}^2\right)\psi\sigma_k^2}{\left(1-\rho_{kx}^2\right)\psi\sigma_k^2 - af''(k)}\right]; \qquad \frac{\partial \omega}{\partial k} = \frac{f''(k)}{\left(1-\rho_{kx}^2\right)\psi\sigma_k^2} < 0;$$

Which show that in order to maintain the optimality criteria, optimal aid inflows must increase with financial wealth, confirming that economies with greater long-term wealth require larger volumes of aid in times of distress. Diminishing returns are essential for changes in capital stock to impact on aid dependence. In linear technologies, with $f''(k) = 0$, changes in capital stock do not affect aid dependence. If diminishing returns are in action, the right direction of variation of aid dependence with capital stock depends on the correlation between aid inflows and the home economy. While counter-cyclical aid reduces aid dependence as capital stock increases, pro-cyclical aid increases it. This seemingly non-sensical effect of pro-procyclical aid if met with diminishing returns results from that in the mean-variance approach, while consumers seek to maximise the mean of their incomes, risk aversion forces them to seek to minimise income variance. When aid is pro-cyclical, its variance becomes less relevant than its mean.

The impact of changes in financial wealth on portfolios gives some idea of how these portfolios will change due to aid inflows. As financial wealth increases, diminishing returns cause capital to increase at a lower rate than its proportion on wealth portfolios. In fact, the proportion of capital decreases as capital stock increases. Hence, if the purpose is to increase investment when the recipient economy has a closed capital account, the best policy is to avoid setting diminishing returns in action.

4.2 The Impact of Perfect Capital Mobility on Optimal Aid Dependence

We have established that with closed capital accounts, aid inflows boost domestic investment, but diminishing returns may reduce the proportion of aid channelled to investment. Does aid policy change in open capital accounts? If apart from holding domestic assets, aid recipients are allowed to hold foreign assets, their optimal portfolio and aid dependence become:

$$[16.a] \quad \omega = \frac{1}{\upsilon\psi}\left[\left(\frac{1-\rho_{k^*k}^2}{\sigma_k}\right)\left(\frac{f'(k)-r}{\sigma_k}\right)+\left(\frac{\rho_{k^*x}\rho_{kx}-\rho_{k^*k}}{\sigma_k}\right)\left(\frac{r^*-r}{\sigma_{k^*}}\right)+\left(\frac{\rho_{k^*k}\rho_{k^*x}-\rho_{kx}}{\sigma_k}\right)\left(\frac{1}{\sigma_x}\right)\right]$$

$$[16.b] \quad \omega^* = \frac{1}{\upsilon\psi}\left[\left(\frac{\rho_{kx}\rho_{k^*x}-\rho_{k^*k}}{\sigma_{k^*}}\right)\left(\frac{f'(k)-r}{\sigma_k}\right)+\left(\frac{1-\rho_{kx}^2}{\sigma_{k^*}}\right)\left(\frac{r^*-r}{\sigma_{k^*}}\right)+\left(\frac{\rho_{k^*k}\rho_{kx}-\rho_{k^*x}}{\sigma_{k^*}}\right)\left(\frac{1}{\sigma_x}\right)\right]$$

$$[16.c] \quad \frac{x}{a} = \frac{1}{\upsilon\psi}\left[\left(\frac{\rho_{k^*k}\rho_{k^*x}-\rho_{kx}}{\sigma_x}\right)\left(\frac{f'(k)-r}{\sigma_k}\right)+\left(\frac{\rho_{k^*k}\rho_{kx}-\rho_{k^*x}}{\sigma_x}\right)\left(\frac{r^*-r}{\sigma_{k^*}}\right)+\left(\frac{1-\rho_{k^*k}^2}{\sigma_x}\right)\left(\frac{1}{\sigma_x}\right)\right]$$

where $\upsilon \equiv 1+2\rho_{k^*k}\rho_{kx}\rho_{k^*x}-\left(\rho_{k^*k}^2+\rho_{kx}^2+\rho_{k^*x}^2\right)$. Hence, the impact of opening the capital account on the holding of domestic securities depends on the correlation between the home economy and the world economy. If these economies are in the same business cycle ($\rho_{k^*k} > 0$) risk-averse investors gain little from diversifying their portfolios. Greater volatility in the international markets makes domestic securities more preferable. However a negative correlation between domestic returns and returns from investment abroad enhances the holding of both domestic and foreign assets.

Assuming $\rho_{k^*k}^2 < 1$, optimal aid dependence decreases with aid volatility, other things held constant. It also depends on the correlation between aid inflows and returns from alternative investment opportunities. In the case of counter-cyclical aid, optimal aid dependence is decreasing in volatility of domestic returns but increasing in growth prospects given by the excess of marginal product of capital to the risk free rate of return. If aid flows follow the cycle of donor economies ($0 < \rho_{k^*x} < 1$), optimal aid dependence decreases with excess returns and decreases with volatility in the international financial markets. The explanation to this lies on the fact that residents would invest on international securities and insure themselves instead of only waiting for aid.

At any given level of risk aversion, it is not the level of aid that matters in the decisions of optimal portfolios, but its volatility. If negatively correlated with domestic returns, aid volatility reduces the holdings of domestic capital on wealth portfolios. If positively correlated with returns from international markets (in which case the rest of the world only helps in good its good times) aid volatility can actually encourage the holding of foreign capital. As said above, this correlation between returns in the international financial markets and aid inflows makes investors indifferent between investing in these markets and waiting for aid in order to insure themselves. This leads to the proposition that aid dependent economies became so because their capital accounts were closed in the first place. Otherwise, diversification would have enabled residents to avoid excessive exposure to investment risk at home.

From [16.*a*-*c*] we can derive the impacts of chances in capital stock on wealth portfolios and aid dependence:

$$[17.a] \quad \frac{\partial \omega}{\partial k} = \left(\frac{1 - \rho_{k^*k}^2}{\upsilon \psi \sigma_k^2} \right) f''(k) < 0$$

$$[17.b] \quad \frac{\partial \omega^*}{\partial k} = \left(\frac{\rho_{kx}\rho_{k^*x} - \rho_{k^*k}}{\upsilon \psi \sigma_k \sigma_{k^*}} \right) f''(k)$$

$$[17.c] \quad \frac{\partial \left(x/a \right)}{\partial k} = \left(\frac{\rho_{k^*k}\rho_{k^*x} - \rho_{kx}}{\upsilon \psi \sigma_k \sigma_x} \right) f''(k)$$

Diminishing returns cause a reduction in the holding of domestic assets as capital stock increases. However, whether this reduction will increase the holding of foreign assets will depend on the strength of the diversification motive. Negative correlation between returns to domestic capital and returns to foreign assets strengthens the possibility that $\rho_{kx}\rho_{k^*x}$ - $\rho_{k^*k} > 0$, in which case [17.*b*] implies that the holding of foreign assets also decreases, as domestic capital increases. On the other hand, since weaker diversification motive means that investors always seek greater returns diminishing returns induce an increased holding of foreign assets when domestic and foreign assets move positively together. A negative correlation between aid inflows and returns to domestic assets is essential to increase the possibility that $\rho_{k^*k}\rho_{k^*k}$ - $\rho_{kx} > 0$ in which case, aid dependence decreases with capital stock.

5. Implications

In the introductory section it was stated that whether there is one-for-one relationship between aid and investment in recipient economies was not the right question. This statement is not to dismiss concerns that under certain conditions aid may not meet its prime objective. Rather, if economic theory — at least under the assumptions made in this paper — predicts that such a one-for-one relationship between aid and investment should not exist, then the question should be: under what conditions the proportion of aid funnelled to investment is maximised? The foregoing discussion suggests two. First, investment risk must be minimal. It is the fear of investment risk that makes risk-averse consumers either refrain from investing or hold most of their wealth in the form of foreign assets. Second, diminishing returns must be weak. This suggestion may sound implausible given that most aid recipient economies are capital-scarce. However, the secret for bright growth prospects lies not on capital mobilisation per se but on the effective use of installed capital. Economies in their early stages of development may also lack human capital, which limits their absorptive capacity to an optimal capital stock that is lower than advanced economies.

There are two related questions on capital-account openness in aid-dependent economies. First, should aid-recipient countries close their capital accounts? One may be tempted to answer 'yes', given that perfect capital mobility generates another leakage, as aid inflows may be funnelled to foreign assets. There is a danger, however, that rational investors may see the introduction of capital control as a signal that something bad is about to happen, if not already happening, about the country's risk rating. If governments were confident to keep investment risk minimal, they would not need capital controls. Hence, the move may actually deter investment instead of promoting it. The second question is: should open economies be aid dependent? The answer depends on the correlation between aid inflows and the performance of wealth invested in the international financial markets. If such a correlation is positive, optimal portfolio choice would enable consumers to insure themselves instead of waiting for aid inflows.

In closing, it is worth pointing out that the results obtained in this paper are not mere theoretical curiosity, but have empirical bearing in actual policy design. At least this is what we can conclude from findings in a large body of empirical literature. For example, Collier and Gunning (1999) report that while aid is peripheral to the group of low-income countries, for Africa the aid-to-gross national product ratio was nearly five times higher than the average of low-income countries as of 1994. Cross-country comparisons show that Africa has held the lowest index of openness to international trade and highest exchange-rate premia in the parallel markets, resulting from exchange-rate controls. Yet available evidence from the 1960s show that Africa is not only the least performing region but also displays the highest volatility. The standard deviation of Africa's GDP during 1961-1994 was only lower than that of the Middle East (Collier and Gunning 1999: Table 5). One of the areas in which East Asia fared better that Africa, explaining better economic performance in the former than the latter, in political stability. There were three times more political assassinations in Africa than in East Asia (Easterly and Levine 1997).

Appendix:

The appendix solves the more general problem of aid dependence with perfect capital mobility, presented in section 4.2. The other cases are easily derived from this solution.

With the possibilities of either holding financial wealth idle or investing it at home and acquiring foreign assets, wealth dynamics becomes:

[A1] $$dW = \left\{a\left[\omega f'(k) + r^* \omega^* + r(1-\omega^*-\omega) + x/a\right] + y - c\right\}dt + +a\left[\omega\sigma_k dz_k + \omega^* \sigma_{k^*} dz_{k^*} + (x/a)\sigma_x dz_x\right]$$

The problem facing consumers is to maximize:

[A2] $$\Lambda\left[s, W(s) - \frac{z(s)}{r+\alpha-\beta}\right] = e^{-\delta s} \max_{c\,\omega\,\omega^*, x/a} E_s\left[\int_s^\infty \frac{e^{-\delta t}}{1-\theta}(c-z)^{1-\theta}\,dt\right]$$

subject to [6], [7] and [A1].

The integral under $E_t[\bullet]$ in the right-hand side of [A2] while depending on the initial values of state variables, it is independent of time:

[A3] $$V\left[W(s) - \frac{z(s)}{r+\alpha-\beta}\right] = \max_{c\,\omega\,\omega^*, x/a} E_t\left[\int_s^\infty \frac{e^{-\delta t}}{1-\theta}(c-z)\,dt\right]$$

(This is the value function used to state problem [9] in the text, with the planning horizon beginning at $s = 0$). We, therefore, express [A2] in the form:

[A4] $$\Lambda\left[t, W(t) - \frac{z(t)}{r+\alpha-\beta}\right] = e^{-\delta t} V\left[W(t) - \frac{z(t)}{r+\alpha-\beta}\right]$$

Dividing the planning horizon at time s, where $0 < s < \infty$:

$$\Lambda\left[0, W(0) - \frac{z(0)}{r+\alpha-\beta}\right] = \max_{\omega\,\omega^*, x/a}\left\{E_0\left[\int_0^s \frac{e^{-\delta t}}{1-\theta}(c-z)^{1-\theta}\,dt\right] + E_s\left[\int_s^\infty \frac{e^{-\delta t}}{1-\theta}(c-z)^{1-\theta}\,dt\right]\right\}$$

The expression under $E_s[\bullet]$ is essentially problem [9], but now starting at time s. Substituting it for Λ and evaluating the problem as $\Delta t = s \rightarrow 0$:

$$\Lambda\left[0, W(0) - \frac{z(0)}{r+\alpha-\beta}\right] = E_0\left[\int_0^s \frac{e^{-\delta t}}{1-\theta}(c-z)^{1-\theta}\,dt\right] + \Lambda\left[s, W(s) - \frac{z(s)}{r+\alpha-\beta}\right]$$
$$\approx \frac{e^{-\delta t}}{1-\theta}(c-z)^{1-\theta}\,dt + \Lambda\left[s, W(s) - \frac{z(s)}{r+\alpha-\beta}\right]$$

Using Itô's lemma to approximate $d\Lambda$:

$$\Lambda\left[0, W(0) - \frac{z(0)}{r+\alpha-\beta}\right] = E_0\left\{\frac{e^{-\delta t}}{1-\theta}(c-z)^{1-\theta}\,dt + \Lambda\left[0, W(0) - \frac{z(0)}{r+\alpha-\beta}\right] + \frac{\partial\Lambda(\bullet)}{\partial t}dt + \right.$$
$$+\frac{\partial\Lambda}{\partial z}(\beta c - \alpha z)dt + \frac{\partial\Lambda}{\partial W}\left[a\left(\omega f'(k) + r^*\omega^* + r(1-\omega^*-\omega) + \tfrac{x}{a}\right) + \right.$$
$$\left. + y - c\right]dt + \frac{a^2}{2}\frac{\partial^2 V}{\partial W^2}\left[(\omega\sigma_k)^2 + (\omega^*\sigma_{k^*})^2 + \left(\tfrac{x}{a}\sigma_x\right)^2 + \right.$$
$$\left.\left. +2\left(\omega^*\omega\rho_{k^*k}\sigma_k\sigma_{k^*} + \tfrac{x}{a}\omega\rho_{kx}\sigma_k\sigma_x + \tfrac{x}{a}\omega^*\rho_{k^*x}\sigma_{k^*}\sigma_x\right)\right]\right\}$$

Rearranging, we get the following Hamilton-Jacobi-Bellman equation:

[A5]
$$-\frac{\partial\Lambda(\bullet)}{\partial t}=\max_{c\,\omega\,\omega^{*},x/a}\left\{\frac{e^{-\delta t}}{1-\theta}(c-z)^{1-\theta}+\frac{\partial\Lambda}{\partial z}(\beta c-\alpha z)dt+\frac{\partial\Lambda}{\partial W}\Big[a\big(\omega f'(k)+r^{*}\omega^{*}+r(1-\omega^{*}-\omega)+\tfrac{x}{a}\big)+y-c\Big]+\frac{a^{2}}{2}\frac{\partial^{2}\Lambda}{\partial W^{2}}\Big[(\omega\sigma_{k})^{2}+(\omega^{*}\sigma_{k^{*}})^{2}+\big(\tfrac{x}{a}\sigma_{x}\big)^{2}+2\big(\omega^{*}\omega\rho_{k^{*}k}\sigma_{k}\sigma_{k^{*}}+\tfrac{x}{a}\omega\rho_{kx}\sigma_{k}\sigma_{x}+\tfrac{x}{a}\omega^{*}\rho_{k^{*}x}\sigma_{k^{*}}\sigma_{x}\big)\Big]\right\}$$

Taking the respective derivatives of [A4] and substituting them into [A5], we get the following current value function:

[A6]
$$\delta V=\max_{c\,\omega\,\omega^{*},x/a}\left\{\frac{e^{-\delta t}}{1-\theta}(c-z)^{1-\theta}+\frac{\partial V}{\partial z}(\beta c-\alpha z)dt+\frac{\partial V}{\partial W}\Big[a\big(\omega f'(k)+r^{*}\omega^{*}+r(1-\omega^{*}-\omega)+\tfrac{x}{a}\big)+y-c\Big]+\frac{a^{2}}{2}\frac{\partial^{2}V}{\partial W^{2}}\Big[(\omega\sigma_{k})^{2}+(\omega^{*}\sigma_{k^{*}})^{2}+\big(\tfrac{x}{a}\sigma_{x}\big)^{2}+2\big(\omega^{*}\omega\rho_{k^{*}k}\sigma_{k}\sigma_{k^{*}}+\tfrac{x}{a}\omega\rho_{kx}\sigma_{k}\sigma_{x}+\tfrac{x}{a}\omega^{*}\rho_{k^{*}x}\sigma_{k^{*}}\sigma_{x}\big)\Big]\right\}$$

Differentiating with respect to c, ω, ω^*, and x/a we get [11.*a*] and [16.*a*-*c*], respectively, in the text. The special cases are derived evaluating at $\rho_{ij}=0$. For example, an aid recipient economy with closed capital account has $\rho_{k^*k}=\rho_{k^*x}=0$.

In order to obtain a closed form-solution of optimal consumption path, we tried solution [12]. Taking the required derivatives and substituting them into [11.*a*] we get:

[A7]
$$c=z+\left[A\left(\frac{r+\alpha}{r+\alpha-\beta}\right)\right]^{-1/\theta}\left(W-\frac{z}{r+\alpha-\beta}\right)$$

Further, taking the respective derivatives of [11.*a*] and substitute them into [A6], using [16.*a*-*c*] as well as [A7] we can derive the marginal propensity to consume:

$$\phi=\left[A\left(\frac{r+\alpha}{r+\alpha-\beta}\right)\right]^{-1/\theta}=\left(\frac{r+\alpha}{r+\alpha-\beta}\right)\left\{\frac{r}{\theta}-(1-\theta)\left[\frac{r}{\theta}+\frac{\Xi}{2\upsilon\theta^{2}}\right]\right\}$$

where :

$$\Xi\equiv\left(1-\rho_{k^*k}^{2}\right)\left[\frac{f'(k)-r}{\sigma_{k}}\right]^{2}+\left(1-\rho_{kx}^{2}\right)\left(\frac{r^{*}-r}{\sigma_{k^*}}\right)^{2}+\frac{1-\rho_{k^*k}^{2}}{\sigma_{x}^{2}}+2\left\{\left(\rho_{k^*x}\rho_{kx}-\rho_{k^*k}\right)\left(\frac{r^{*}-r}{\sigma_{k^*}}\right)\left[\frac{f'(k)-r}{\sigma_{k}}\right]+\left(\frac{\rho_{k^*k}\rho_{k^*x}-\rho_{kx}}{\sigma_{x}}\right)\left[\frac{f'(k)-r}{\sigma_{k}}\right]+\left(\frac{\rho_{k^*k}\rho_{kx}-\rho_{k^*x}}{\sigma_{x}}\right)\left(\frac{r^{*}-r}{\sigma_{k^*}}\right)\right\}$$

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