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**MEASURING DEVELOPMENT? A STATISTICAL  
CRITIQUE OF THE UNDP'S HUMAN  
DEVELOPMENT INDEX**

Mark McGillivray and Howard White

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Measuring Development? A Statistical Critique of  
the UNDP's Human Development Index

Mark McGillivray  
Deakin University  
Geelong, Australia  
and

Howard White  
Institute of Social Studies  
The Hague, The Netherlands

ABSTRACT

In its *Human Development Report 1990*, the UNDP proposed its now well-known Human Development Index (HDI). A subsequent version of this index appeared in the UNDP's *1991 Report*. The index is a composite indicator of development, comprising measures of longevity, educational attainment and material living standards. This paper critically examines both versions of the HDI, concluding that its treatment of income is inappropriate; its year-on-year comparability is limited; it is robust with respect to measurement error; and that its contribution to the assessment of development levels differs markedly among country groups.

I INTRODUCTION

The release of the UNDP's annual *Human Development Report* series is one of the major development initiatives of the early 1990s. The UNDP has not only provided an alternative to the World Bank's *World Development Report* series, but a timely reminder of reassessment of development strategies that took place in the 1970s. As is well-known, this period saw widespread questioning of the perceived excessive pre-occupation with income measures, especially GNP per capita, with researchers and practitioners alike emphasising broader social and political indicators of development. The UNDP has attempted to operationalise these broader ideological principles by proposing and reporting values

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of its now well known "Human Development Index" (HDI), which measures inter-country development levels based on life expectancy, educational attainment and material living standards.

This paper critically appraises the HDI as it appears in the 1990 and 1991 issues of the *Human Development Report*. This appraisal is conducted at a statistical level and does not question the ideological basis of the HDI.<sup>1</sup> Our appraisal commences in Section II with a presentation of the index and details of its construction. We highlight developments in the index, especially its treatment of income. The discussion of income leads in Section III to an analysis of the disparity between different countries compared on the basis of income and the HDI. In Section IV we *inter alia* take the UNDP to task regarding its assertion the HDI ranks countries very differently to per capita income. That this was not the case was a criticism of previous measures as discussed in previous studies (see, for example, Larson and Wilford (1979) and Hicks and Streeten (1979)). The methodology of these studies is extended and applied to the HDI in Section IV. To the extent that there are differences in rankings, we further consider in Section V if these are simply the result of measurement error. Section VI examines a number of possible refinements to the index and concluding remarks are made in Section VII.

## II THE HUMAN DEVELOPMENT INDEX

The 1990 HDI was defined as:

$$HDI_j = 1 - \left( \frac{1}{3} \sum_{i=1}^3 I_{ij} \right) \quad (1)$$

where  $I_{ij}$  is the  $i$ th indicator of human deprivation in country  $j$ ,  $i = 1, 2$  and  $3$  and  $j = 1, \dots, 1$ . Country  $j$ 's deprivation is assessed on the basis of the following variables: life expectancy ( $X_{1j}$ ) in 1987, adult literacy ( $X_{2j}$ ) in 1985 and the logarithm of purchasing power adjusted GDP per capita ( $X_{3j}$ ) in 1987. The latter serves as a proxy for 'income for a decent standard of living' (UNDP, 1990, p.13). Deprivation is treated as a relative concept

by scaling these variables within the range of zero to one using the equation:

$$I_{ij} = \left( \frac{(X_i^* - X_{ij})}{(X_i^* - X_i^m)} \right) \quad (2)$$

where  $X_i^*$  is a subjectively chosen desired value of the  $i$ th variable,  $X_{ij}$  is the value of the  $i$ th variable for country  $j$  and  $X_i^m$  is the observed minimum. If a country attains the desired value of the  $i$ th variable there is no relative deprivation ( $I_{ij} = 0$ ). If the  $j$ th country achieves the desired values of each of the  $X_i$ s, then the  $I_{ij}$ s are all zero and the value of the HDI takes its maximum value of one. Conversely, the greater is the gap between the three  $X_i^*$ s and the corresponding  $X_{ij}$ s of the  $j$ th country, the lower is the value of that country's HDI. The index reaches its minimum value of zero if a country has  $X_i^m$  for each of the  $X_i$ s.

In calculating HDI values of 130 countries (those with populations of at least one million people) the UNDP's desired value for life expectancy is 78 years, which is that achieved by Japan in 1987, while that for adult literacy is a rate of 100 percent. For purchasing power adjusted per capita GDP, the corresponding value is the logarithm of \$4,861, which was the average official poverty line income in nine industrial countries. The 43 countries whose per capita GDPs exceeded this amount was "capped" to be \$4,861. This treatment of income is discussed below.

The 1990 HDI was the subject of a number of refinements in the *Human Development Report 1991*. The resulting index, the 1991 version of the HDI, differed from its predecessor in two main respects.<sup>2</sup> Firstly, literacy was replaced with "educational attainment", by redefining  $X_2$  as follows:

$$X_{2j} = a_1 LIT_j + a_2 YRS_j, \quad (3)$$

where  $a_1$  and  $a_2$  are weights set at 2/3 and 1/3 respectively,  $LIT$  is the adult literacy rate in 1985 and  $YRS$  is mean years of schooling received per person aged 25 years and over in 1980. Thus defined, the desired level of educational attainment

corresponded to an observed maximum of 70.1 (that achieved by the United States).

The second major refinement was made to the treatment of income. The observed range of GDPs per capita were divided in multiples of the poverty line  $y^*$  as follows:

$$\begin{aligned}
 X_{3j} &= y_j && \text{for } 0 < y_j \leq y^* \\
 &= y^* + 2(y_j - y^*)^{1/2} && \text{for } y^* \leq y_j \leq 2y^* \\
 &= y^* + 2(y^*)^{1/2} + 3(y_j - 2y^*)^{1/3} && \text{for } 2y^* \leq y_j \leq 3y^* \quad (4) \\
 &= y^* + 2(y^*)^{1/2} + 3(y^*)^{1/3} + 4(y_j - 3y^*)^{1/4} && \text{for } 3y^* \leq y_j \leq 4y^* \\
 &\text{and so on,}
 \end{aligned}$$

where  $y_j$  is country  $j$ 's real GDP per capita and  $y^*$  is a poverty line real GDP per capita of \$US4,829. The 1990 HDI formulation by comparison was:

$$\begin{aligned}
 X_{3j} &= \log y && \text{for } 0 < y \leq y^* \\
 X_{3j} &= \log y^* && \text{for } y > y^*.
 \end{aligned} \tag{5}$$

Unlike the 1990 formulation, the 1991 formulation therefore essentially allows  $X_{3j}$  to rise marginally (although, as shown below, extremely so) after exceeding the poverty line.

HDI's for selected countries are shown in Table 1. In accordance with the UNDP's practice, these countries are divided into 3 sub-groups: high human development countries (those with HDI values equal to or greater than 0.800), medium human development countries (those with HDI values within the range of 0.500 to 0.799) and low human development countries (those with HDI values less than 0.500). A number of countries listed in Table 1 experienced often substantial falls in rankings (for example, Sierra Leone, from 127 in 1990 to 160 in 1991). This is primarily due to the inclusion of all countries in base HDI calculations in the UNDP's 1991 data tables, whereas the 1991 *Report* listed the HDI's of countries with populations of less than 1 million separately.<sup>3</sup> As discussed in more detail below, the HDI's between years are not comparable both because of redefinition and because of a non-constant scaling factor.



Table 1  
HDI Profile for Selected Countries

Country	1990 HDI		1991 HDI	
	Value	Rank	Value	Rank
High Human Development Countries				
Japan	0.996	1	0.993	1
United Kingdom	0.970	10	0.967	11
U.S.A.	0.961	19	0.976	7
Argentina	0.910	32	0.854	43
Malaysia	0.800	46	0.802	52
Medium Human Development Countries				
South Africa	0.731	63	0.766	57
Saudi Arabia	0.702	67	0.697	69
Philippines	0.714	66	0.613	84
Botswana	0.646	73	0.524	95
Low Human Development Countries				
Morocco	0.489	87	0.431	108
India	0.439	94	0.308	123
Bangladesh	0.318	108	0.186	136
Mozambique	0.239	118	0.155	146
Niger	0.116	130	0.079	155
Sierra Leone	0.150	127	0.048	160

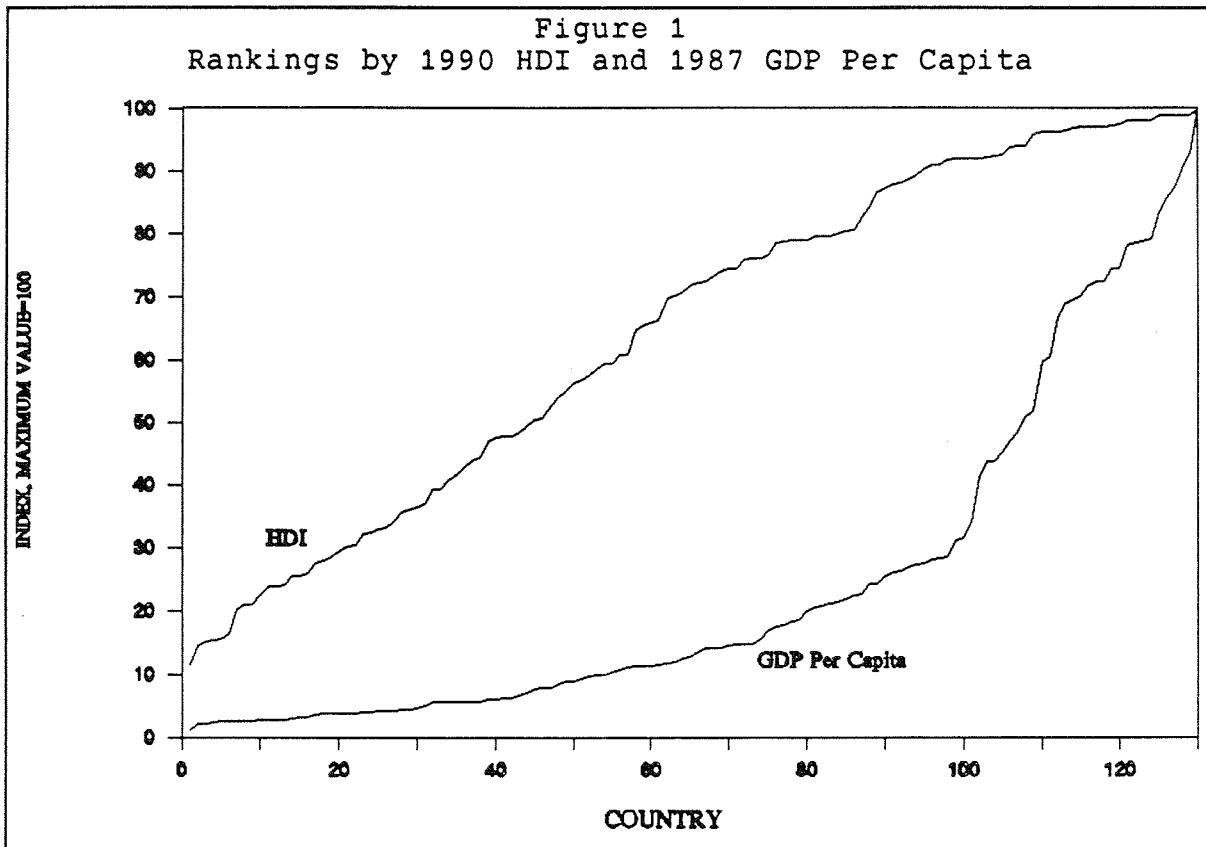
Source: UNDP (1990, 1991)

### III WHY THE DISPARITY BETWEEN THE HDI AND INCOME PER CAPITA?

The 1990 *Report* makes two claims in particular for the HDI: (i) that it ranks countries very differently to income measures; and (ii) that the disparity between countries is less for the HDI than it is for income per capita. The first of these is discussed below. Here we consider the latter and the related issue of the treatment of income in constructing the HDI.

The 1990 *Report* contains a graph headed "Ranking of countries' GNP per capita and HDI". This graph, which is a recurring motif throughout the *Report*, is reproduced as Figure 1.<sup>4</sup> What does this graph show? It is simply a graph of HDI and GDP per capita scores (indexed so that the highest value equals 100 in our example) each sorted separately in ascending order. There is no information on how country rankings compare by the two criteria. The graph is accompanied in the *Report* by the following blurb:

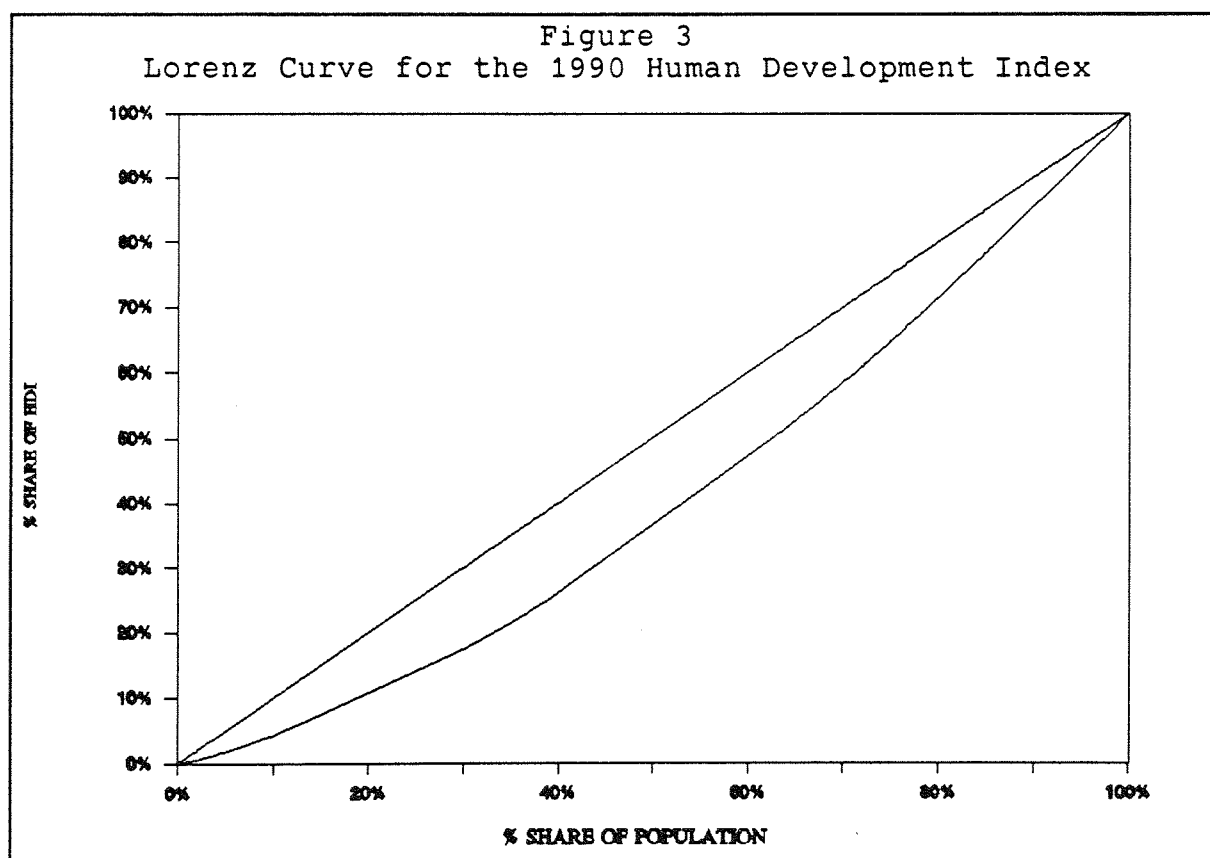
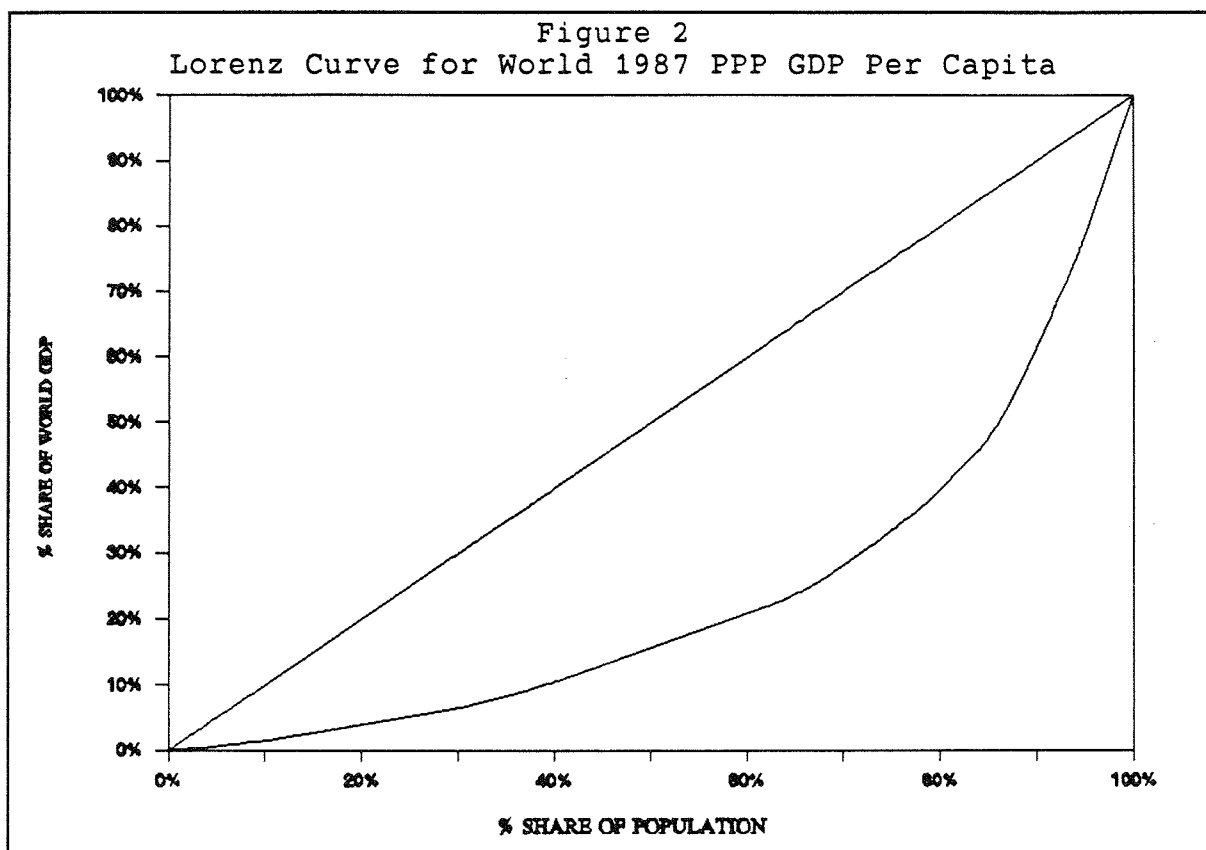
"The chart shows two separate distributions of countries.



The upper curve shows their ranking according to the human development index while the lower curve shows their ranking according to GNP per capita. The two curves reveal that the disparity among countries is much greater in income than in human development. There is no automatic link between the level of per capita income in a country and the level of its development" (UNDP 1990, p.15).

As already explained, this last statement cannot be verified from the graph, since the two indices are ranked separately. But we can see that there are many countries with relatively low per capita income levels compared to a few (the top 20% or so) with much higher income - by contrast the HDI rises steadily over the range of countries. Thus it is correct that disparities in the HDI are less than those in GDP per capita. Figure 1 is of course not a Lorenz Curve (though it looks very much like one). Using the data provided in the *Report*, Lorenz curves were drawn for world income distribution and the international distribution of the HDI.<sup>5</sup> These graphs are shown as Figures 2 and 3 respectively. The corresponding gini coefficients are 0.53 and 0.34.<sup>6</sup>

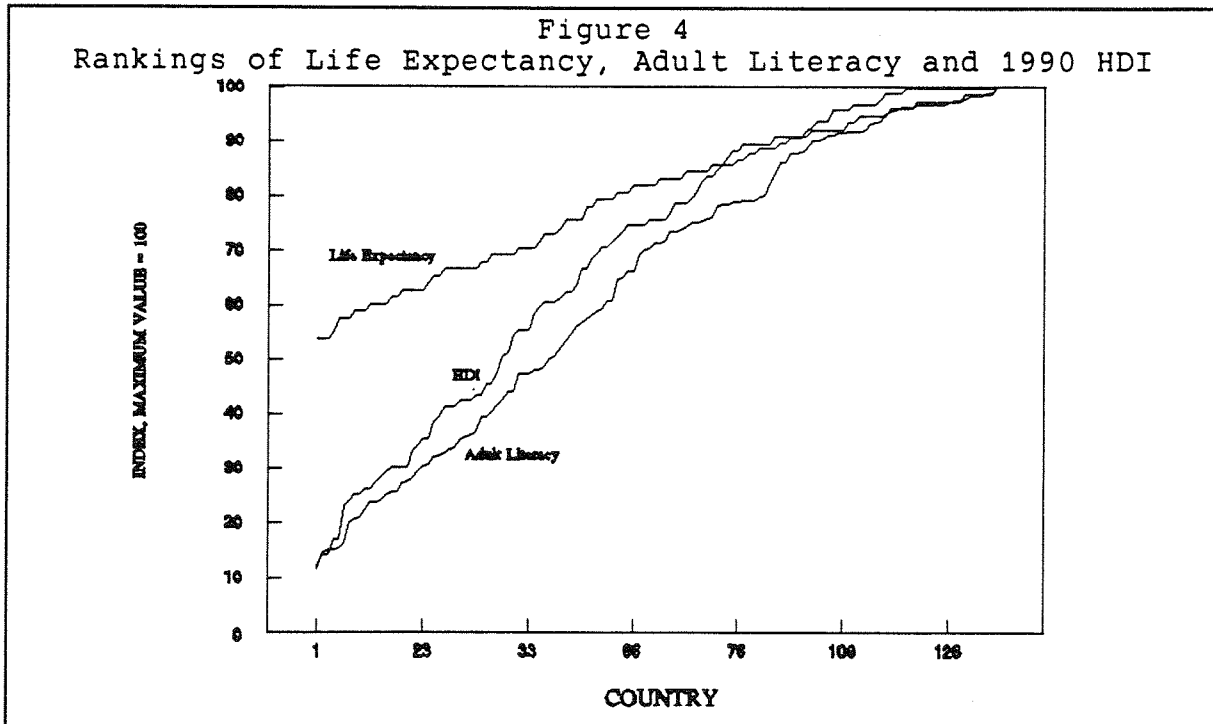
The question then arises, what is the source of this disparity? There are two potential sources. First, the



inclusion of life expectation and literacy. Second, the treatment of income in constructing the HDI. Physiological limits on life expectancy and a 100% ceiling on literacy restrict the disparity that may be displayed by these variables. A

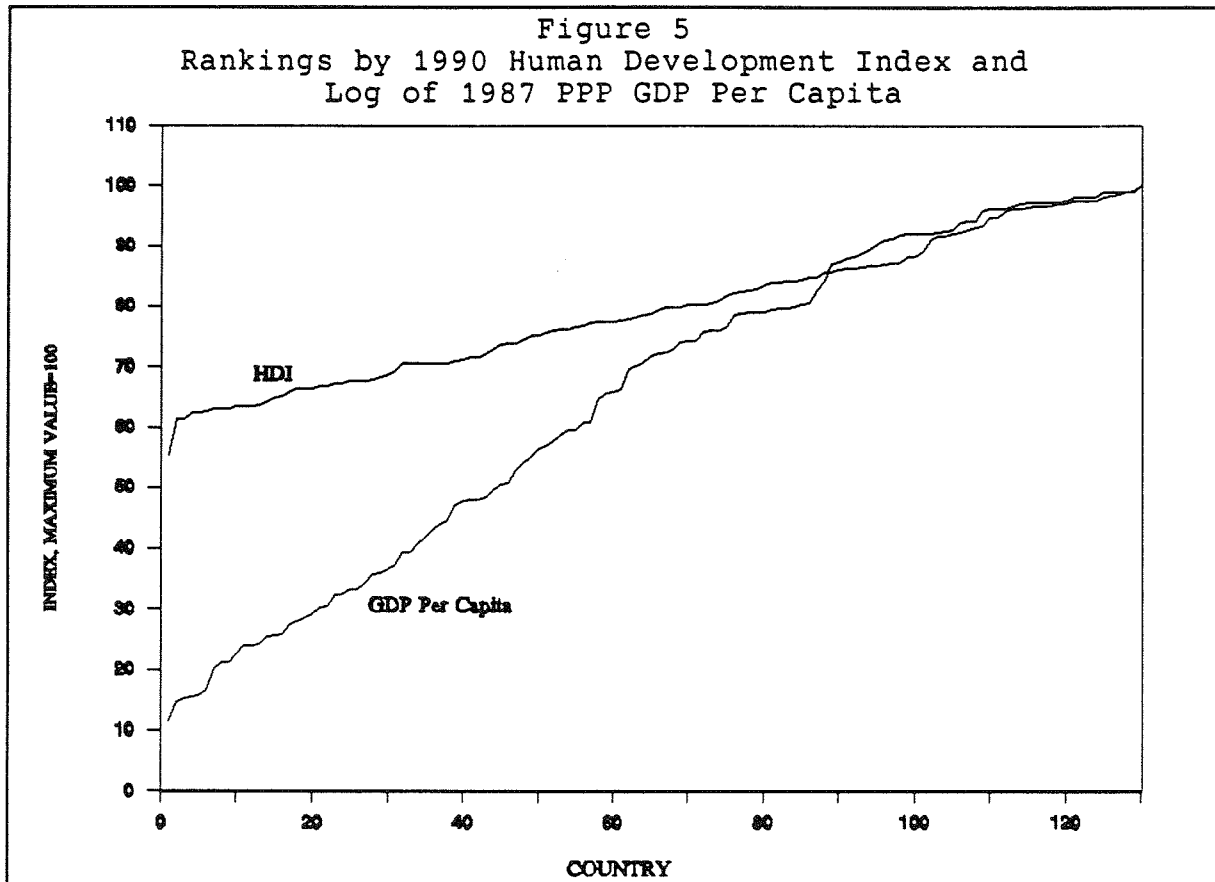
ranking of them, analogous to that in Figure 1, is shown in Figure 4. The inclusion of these variables will clearly reduce disparity (though whether or not they affect ranking will depend on their correlation with per capita income).

There are two adjustments made to income in the 1990 *Report*. The first is to take logs. A log transformation of income is appealing to economists, since it captures diminishing marginal utility of income - that is successive increments in income give successively smaller increments in utility. Such a transformation closes the gap at the top end - in a way that makes multiples based on log figures correspond more closely to perceptions about relative welfare levels in the different



countries. As Hamilton (1990, p.161) points out, the gap in actual GNP per capita between Australia and Norway is quite a bit larger than that between Taiwan and Ethiopia - but surely our perception of the development gap between the two is the opposite of this. Making a log transformation gives this result. Because of this "closing of the gap" the disparity in log income per capita is much less than that in income per capita - and indeed less than that in the HDI. Figure 5 shows the graph of countries ranked by HDI and log GDP per capita and Figure 6 shows the

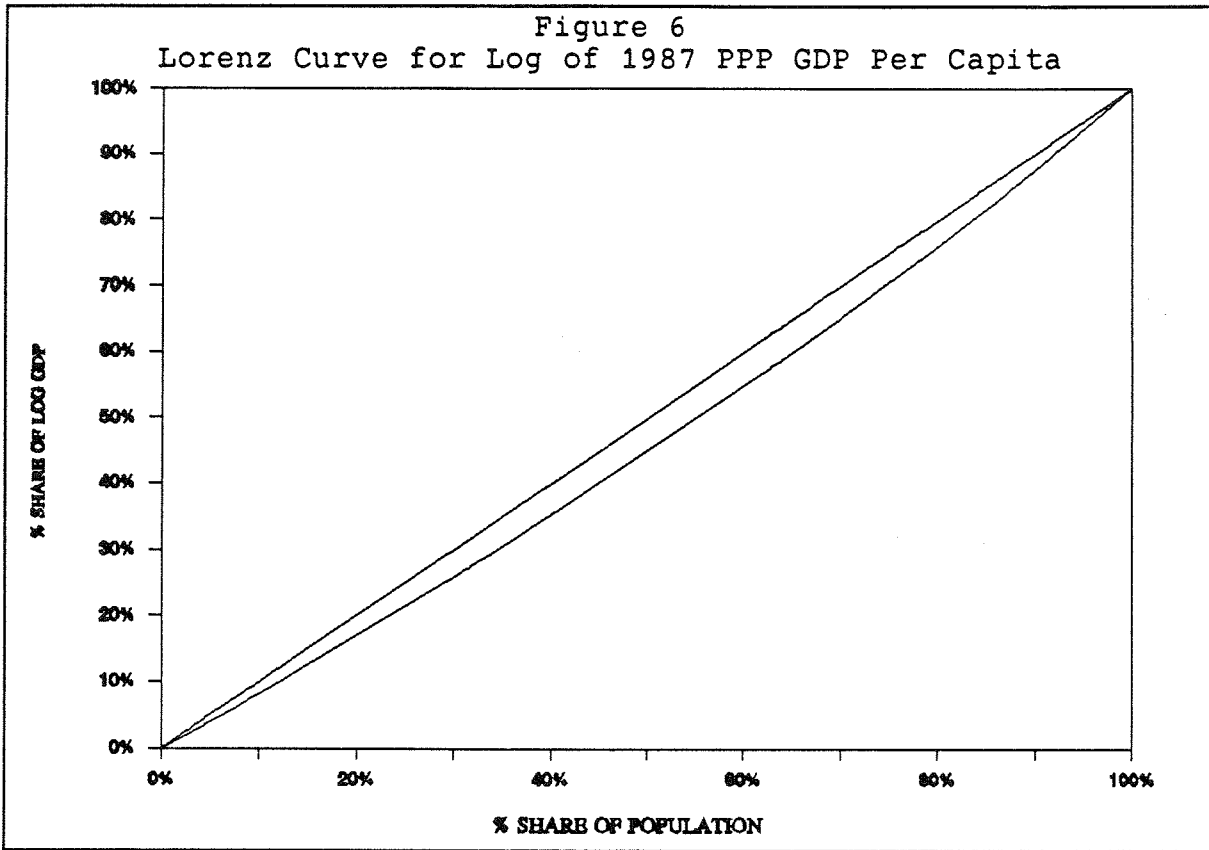
Lorenz curve for world log income - the gini in this case is only 0.07 The above quotation should perhaps be amended to read:



"The curves reveal that the disparity among countries is much greater in income than in human development and greater still than that in the log of per capita income".

The use of logs, which appears perfectly acceptable, also plays a major role in making the HDI have less disparity than GDP per capita.

Yet the 1990 *Report* does not stop there. It goes on to impose an arbitrary ceiling (the average poverty line developed countries) above which increments in income are assumed to have no value to development whatsoever. A graphical representation of this treatment is given in Figure 7. Up to the cut-off of \$4,861, the log value of GDP per capita is used in calculating the index. At any higher income level the log of \$4,861 is used. This appears a difficult position to support - would the authors of the *Report* accept a marginal rate of taxation of 100% above an income level of \$4,861 since this apparently will not damage



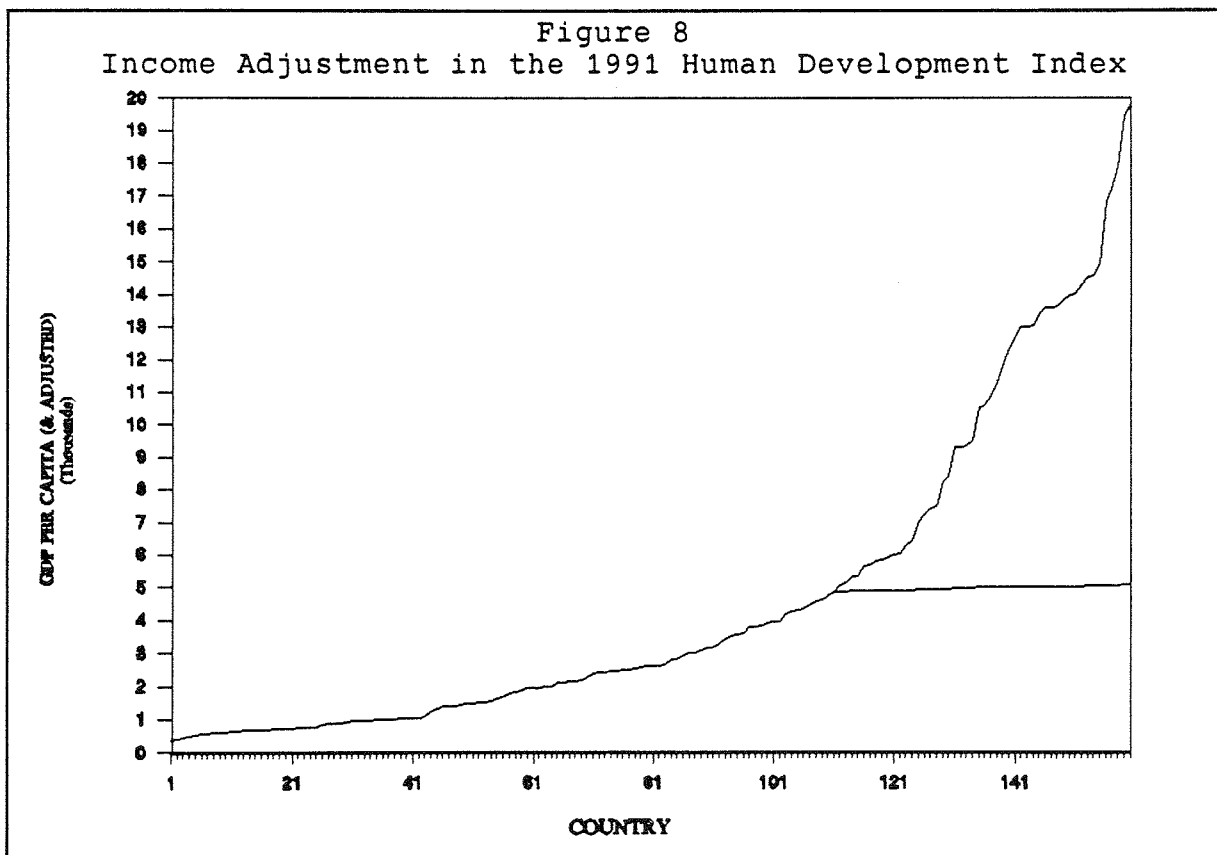
their welfare?

This point was taken on board in the 1991 *Report* which made the following change in the construction of the HDI:

"The idea of diminishing returns to income is now

better captured be giving a progressively lower weight to income beyond the poverty cut-off point, rather than the zero weight previously given. The zero weight was found to be too drastic an adjustment, particularly for higher income societies" (UNDP 1991, p.15).

In the new treatment, income below the poverty threshold was taken at its actual value, above that successive roots (square root etc., as shown in equation 4) were taken for each multiple of the poverty level. The impact of this is shown in Figure 8.<sup>7</sup> The cut-off is practically the horizontal one of the previous year. The practice adopted values the first \$4,829 at its actual value and the second increment at only \$139 - the 100% tax rate has been replaced with one of 97%!



One of the problems with the capping procedure is the resulting sensitivity of higher income countries (those above the capping level) to small deviations in the other two indicators. This has been pointed out by Pomfret:

"Amongst the highest ranked countries, trivial variations in life expectancy and literacy dominate the HDI rankings, placing Ireland above the United States. However praiseworthy Ireland's society, it is paradoxical to rank it above the United States on any scale purporting to assess economic development. The HDI appears to need further refinement before it corresponds to development-oriented LDC government's targets or to the aspirations of most people, which

seem more heavily weighted towards the desire for increased material wealth" (1992, p.9).

We would argue that the capping of income in any form should be abandoned - the use of log GDP per capita is a theoretically appealing and practically transparent alternative. A defence of the UNDP's practice may be found in Technical Notes 1 of the 1991 *Report* ("The human development index - key components and robustness"), in which the Spearman's rank correlation of the HDI with the HDI constructed with different treatments of income are reported. These correlations are all very high - the lowest being 0.994 if unlogged income per capita is used with no cut-off. We would expect such high correlations to be observed: if, as we have argued, the different components of the index are highly correlated then excluding one of them (even if only over a range of countries, as the capping procedure does) will not greatly alter the rankings.<sup>8</sup> Two conclusions may therefore be drawn from the UNDP's own analysis of the different treatment of income: (i) they add further support to the position that income is a good proxy for the other variables; and (ii) if a simple treatment yields the same result as a more complex one then why not adopt the simple treatment?

#### IV THE HDI: AN INSIGHTFUL NEW INDICATOR?

We now consider the issue of the HDI's contribution to the assessment of inter-country development levels. The UNDP has claimed that the "human development index ranks countries very differently from the way GNP per capita ranks them" (UNDP, 1990, p.14). This claim was made on the basis often very large differences between HDI and GNP per capita ranks.<sup>9</sup> By making such claims, the UNDP effectively invites scrutiny of its indexes' contribution.

A number of previous studies have assessed predecessors of the HDI on this criterion. For example, Larson and Wilford (1979), using simple correlation coefficient analysis, argued that the "Physical Quality of Life Index" (PQLI, a composite of life expectancy, adult literacy and infant mortality)<sup>10</sup> was



redundant in the sense that it ranked countries in a "statistically indistinguishable fashion" to GNP per capita (Larson and Wilford, 1979, p. 583). Similar exercises have been conducted by Hicks and Streeten (1979), who looked *inter alia* at the "Development Index" (a composite of 18 social and economic indicators)<sup>11</sup> and McGillivray (1991), who looked at the 1990 HDI.

A problem with these studies is ambiguity over the extent of statistical association, as measured by a correlation coefficient, that actually determines one variable redundant with respect to another.<sup>12</sup> It is necessary to specify some arbitrary threshold coefficient value which differentiates redundancy from non-redundancy. In this paper, we opt for two thresholds. The first of 0.90 and the second of 0.70. If the rank correlation coefficient between the HDI and an already used (or "pre-existing") indicator (in this case GNP per capita) is not significantly less than 0.90 we call this "Level 1" redundancy. If it is not significantly less than 0.70 we call this "Level 2" redundancy. Clearly, the second of these tests is the more difficult for the HDI to pass in terms of not being redundant.

Why have we chosen these levels? We believe that 0.90 is sufficiently high to say that if two variables have a correlation of this order of magnitude then it is difficult to claim that one is imparting additional information to that given by the first. The lower threshold is used as that suggested by existing studies. Whilst such studies have not explicitly defined a threshold, they have had implicit the notion that a new index is redundant if the majority of its variation is accounted for by any one of the pre-existing indicators. This can be formerly interpreted as test that the coefficient of variation be not less than 0.50, which we convert to a correlation coefficient of 0.70.<sup>13</sup>

A related question concerns the redundancy of the HDI *vis-a-vis* its own components since they are also, by definition, pre-existing indicators. An obvious starting-point in investigating

this issue would be to correlate the complete HDI on its component variables individually. This is though an overly demanding test. It involves correlating the HDI partially on itself, with a high correlation coefficient the inevitable result, especially given the equal weighting assigned to each component. With these considerations in mind, we opted to first decompose the index by calculating its values based on two of its three components only, and then correlating these values with the excluded component. This not only tests the redundancy of the index *vis-a-vis* the excluded component, both with and without this component, but also the composition of the HDI. If the component is correlated at the chosen threshold(s), then its inclusion provides few additional insights to the assessment of development levels. For reasons of parsimony, it would be better to base the index on the two included components or, better still, on the excluded component alone.

Results are reported in Tables 2 to 5. Correlation coefficients were calculated for total samples of 130 countries for 1990 and 160 for 1991. Coefficients for sub-samples of countries comprising those countries classified as industrial, developing, low human development (LHD), medium human development (MHD) and high human development (HHD) were also calculated. Each of the single HDI components referred to in Tables 2 to 5 were scaled according to the procedure described by equation (2) and then subtracted from one. The exception to this were the GDPs per capita of the industrial countries comprising the 1990 HDI. All but five of these countries GDPs per capita were in excess of the poverty line  $y^*$ , and given the UNDP's previously discussed treatment, were set equal to zero. For these countries, the inverse of actual logarithmic rather than scaled logarithmic GDPs per capita were used.

First consider the estimated correlation coefficients between HDI components, as shown in Tables 2 and 3. In these tables,  $HDI_{2,3}$  refers to a composite of adult literacy/educational attainment ( $X_2$ ) and adjusted GDP per capita ( $X_3$ ),  $HDI_{1,3}$  refers to

Table 2  
Spearman Rank-order Correlation Coefficients Between 1990 HDI Components

Variables	All Countries (n = 130)	Developing Countries (n = 99)	Industrial Countries (n = 31)	Low Human Development Countries (n = 44)	Medium Human Development Countries (n = 40)	High Human Development Countries (n = 46)
HDI <sub>1,2,3</sub> & Life Expectancy	0.827*	0.820*	0.930*	0.584**	0.447**	0.612*
HDI <sub>1,2,3</sub> & Adult Literacy	0.847*	0.609*	0.844*	0.187	0.056	0.481**
HDI <sub>1,2,3</sub> & Adjusted GDP per capita	0.932*	0.654**	0.826*	0.127	0.051	0.258
Life Expectancy & Adult Literacy	0.867*	0.767*	0.283**	0.397**	0.246	0.396**
Life Expectancy & Adjusted GDP per capita	0.859*	0.831*	0.735*	0.378**	0.299	0.355**
Adult Literacy & Adjusted GDP per capita	0.743**	0.607**	0.517*	-0.086	-0.255	0.196

\*: Level 1 redundancy. \*\*: Level 2 redundancy. All HDI variables are expressed as one minus their scaled values, with the exception of the GDPs per capita of industrial countries. GDPs per capita were converted to logarithms prior to scaling. GDPs per capita are purchasing power adjusted. All data used in calculation of coefficients taken from UNDP (1990).

Table 3  
Spearman Rank-order Correlation Coefficients Between 1991 HDI Components

Variables	All Countries (n = 160)	Developing Countries (n = 127)	Industrial Countries (n = 33)	Low Human Development Countries (n = 63)	Medium Human Development Countries (n = 44)	High Human Development Countries (n = 53)
HDI <sub>1,2,3</sub> & Life Expectancy	0.927*	0.891*	0.652**	0.826*	0.196	0.605**
HDI <sub>1,3</sub> & Adult Literacy	0.852*	0.762**	0.532**	0.670**	0.195	0.595**
HDI <sub>1,2</sub> & Adjusted GDP per capita	0.842*	0.752**	0.896*	0.381	0.376**	0.680**
Life Expectancy & Adult Literacy	0.881*	0.822*	0.457**	0.741*	0.337**	0.519**
Life Expectancy & Adjusted GDP per capita	0.846*	0.812*	0.778*	0.515**	-0.090	0.660**
Adult Literacy & Adjusted GDP per capita	0.735**	0.645**	0.741*	0.269	-0.421	0.560**

\*: Level 1 redundancy. \*\*: Level 2 redundancy. All HDI variables are expressed as one minus their scaled values. GDPs per capita are purchasing power adjusted. All data used in calculation of coefficients taken from UNDP (1991).

a composite of life expectancy ( $X_1$ ) and adjusted GDP per capita and  $HDI_{1,2}$  refers to a composite of life expectancy and adult literacy/educational attainment. From Table 2 we observe that the 1990 HDI is subject to redundancy of composition with respect to the full, developing and industrial country samples. Indeed, without exception, a restricted HDI is correlated with the excluded component at magnitudes deeming that component's inclusion redundant at either Level 1 or 2. Also, by implication, the entire index is redundant *vis-a-vis* that component. These conclusions are also drawn with respect to the 1991 HDI for these samples based on the coefficients reported in Table 3. Much less consistent results were obtained with the LHD, MHD and HHD sub-samples. While it is generally the case that there is redundancy of composition if one considers the HHD countries only, the reverse seems to be the case with the LHD and MHD countries. Indeed, for the MHD sub-sample, including the life expectancy component in the 1990 HDI and GDP per capita in the 1991 version are the only instances in which redundancy prevails.

Tables 2 and 3 also contain rank-order coefficients between individual components of the HDI. In the overwhelming majority of instances, these coefficients are significant at the previously mentioned levels. These findings serve to emphasise redundancy in the composition of the index: in most cases, not only is adding a third variable to the index is of little further insight, but basing the index on a single component would provide roughly similar results to any one of the others. Though some exceptions ought to be emphasised, each of which are applicable to the MHD countries. In the case of these countries, negative correlation, or at best negligible correlations are exhibited between the adult literacy and GDP per capita components for both 1990 and 1991 HDI versions, and between life expectancy and GDP per capita with the 1991 index. These correlations notwithstanding,  $HDI_{1,2}$  is still correlated at a magnitude deeming the GDP per capita component's inclusion into the index redundant.

Now consider the rank-order correlation coefficients between the HDI and GNP per capita. These are shown in Tables 4 and 5. For both the 1990 and 1991 versions of the HDI, we observe Level 1 redundancy for the total, industrial, developing, and HHD country samples, and Level 2 redundancy each of the remaining samples. Tables 4 and 5 also provide rank-order correlation coefficients between GNP per capita and the HDI components. We observe that in the majority of cases these correlation coefficients are insignificantly different from or greater than either threshold level. This would seem to lie at the heart of the indicated redundancy of the HDI *vis-a-vis* GNP for the above-mentioned samples.

#### V THE HDI AND MEASUREMENT ERROR

We saw in the preceding section that the correlation between the HDI based on any two of its components and the remaining component and with GNP per capita is generally less when considering sub-groupings rather than all countries. This is unsurprising in that small variations in rankings have a greater effect in smaller samples. We now ask whether the results reported above may arise solely on account of measurement error. This also serves to test the robustness of the HDI rankings with respect to measurement error.

The problems in the collection of social and economic data, especially in developing countries, are well known. Thus even if say GNP per capita and HDI are in fact perfectly correlated differences in ranking may emerge simply due to measurement error. This problem is likely to be particularly acute for countries at lower levels of development. Here we investigate whether this is the case by adjusting observed HDI values, as reported by the UNDP, according to a set of random errors to obtain "true" values, and then comparing country rankings accordingly.

In the case of the 1991 HDI, the true index is defined as follows:

Table 4  
Spearman Rank-order Correlation Coefficients Between 1990 HDI Components and GNP per capita

Variables	All Countries (n = 119)	Developing Countries (n = 79)	Industrial Countries (n = 27)	Low Human Development Countries (n = 40)	Medium Human Development Countries (n = 34)	High Human Development Countries (n = 45)
HDI & GNP per capita	0.889*	0.770*	0.856*	0.451**	0.317**	0.785*
Life Expectancy & GNP per capita	0.862*	0.688*	0.734*	0.265	0.161	0.757*
Adult Literacy & GNP per capita	0.754**	0.506*	0.431*	0.062	0.337**	0.594**
Adjusted GDP per capita & GNP per capita	0.913*	0.886*	0.965*	0.684*	0.833*	0.460**

\*: Level 1 redundancy. \*\*: Level 2 redundancy. All HDI variables are expressed as one minus their scaled values. GDPs per capita were converted to logarithms prior to scaling. GDPs per capita are purchasing power adjusted. All data used in calculation of coefficients taken from UNDP (1990).

Table 5  
Spearman Rank-order Correlation Coefficients Between 1991 HDI Components and GNP per capita

Variables	All Countries (n = 160)	Developing Countries (n = 127)	Industrial Countries (n = 33)	Low Human Development Countries (n = 63)	Medium Human Development Countries (n = 44)	High Human Development Countries (n = 53)
HDI & GNP per capita	0.910*	0.854*	0.891*	0.511**	0.464**	0.796*
Life Expectancy & GNP per capita	0.863*	0.784*	0.754*	0.477**	0.065	0.669**
Adult Literacy & GNP per capita	0.767**	0.621**	0.708*	0.287	-0.217	0.619**
Adjusted GDP per capita & GNP per capita	0.954*	0.921*	0.929*	0.686**	0.799*	0.937*

\*: Level 1 redundancy. \*\*: Level 2 redundancy. All HDI variables are expressed as one minus their scaled values. GDPs per capita are purchasing power adjusted. All data used in calculation of coefficients taken from UNDP (1991).



$$HDI_j^\# = 1 - \left( \frac{1}{3} \sum_{i=1}^3 I_{ij}^\# \right) \quad (6)$$

where  $I_{ij}^\#$  is the  $i$ th correctly measured indicator of human deprivation in country  $j$  and as before  $j = 1, \dots, 1$ .  $I_{ij}^\#$  is in turn defined as:

$$I_{ij}^\# = \left( \frac{(X_i^{*\#} - X_{ij}^\#)}{(X_i^{*\#} - X_i^{\#\#})} \right) \quad (7)$$

where  $X_i^{*\#}$  and  $X_i^{\#\#}$  are the correctly measured maximum and minimum values of the  $i$ th variable respectively among 1 countries and  $X_{ij}^\#$  is the true value of the  $i$ th variable for country  $j$ .

For  $i = 1, 2$ , (i.e., for  $X_{1j}$  and  $X_{2j}$ ):

$$X_{ij}^\# = (1 + \epsilon_{ij}) X_{ij} \quad -v \leq \epsilon_{ij} \leq v \quad (8)$$

where  $\epsilon_{ij}$  is a random error ranging from  $-v$  to  $+v$ . For  $i = 3$ , (i.e. for  $X_{3j}$ ):

$$\begin{aligned} X_{3j}^\# &= (1 + \epsilon_{3j}) y_j && \text{for } 0 < (1 + \epsilon_{3j}) y_j \leq y^* \\ &= y^* + 2 \{ (1 + \epsilon_{3j}) y_j - y^* \}^{1/2} && \text{for } y^* \leq (1 + \epsilon_{3j}) y_j \leq 2y^* \\ &= y^* + 2 \{ y^* \}^{1/2} + 3 \{ (1 + \epsilon_{3j}) y_j - 2y^* \}^{1/3} && \text{for } 2y^* \leq (1 + \epsilon_{3j}) y_j \leq 3y^* \\ &= y^* + 2 \{ y^* \}^{1/2} + 3 \{ y^* \}^{1/3} + 4 \{ (1 + \epsilon_{3j}) y_j - 3y^* \}^{1/4} && \text{for } 3y^* \leq (1 + \epsilon_{3j}) y_j \leq 4y^* \\ &\text{and so on,} \end{aligned} \quad (9)$$

where  $\epsilon_{3j}$  is also a random error ranging from  $-v$  to  $+v$ . The same procedure was used to compute true 1990 HDI values. To enable comparative assessments between the 1990 and 1991 versions of the HDI,  $\epsilon_+$  was held constant for each country.<sup>14</sup>

Adjusted HDI values were obtained for each country appearing in the 1990 and 1991 Reports.<sup>15</sup> Results of ranking differences between observed and adjusted HDIs full country samples are respectively shown below in Tables 6 and 7. It was first assumed that the measurement error range was between -10 percent and +10 percent of the observed values of each HDI component for each country. The maximum absolute change any one country's ranking was 24 for both 1990 and 1991 HDIs. Fifty-nine countries' rankings were elevated in 1990 and 77 in 1991, while the reverse was the case for 56 and 75 countries in these years respectively.

Table 6  
Summary Table: Observed versus Adjusted HDI Rankings

Ranking Changes	Random Error Range ( $\nu$ ): +/- 10%									
	1990 HDI					1991 HDI				
	Life Expectancy	Educational Attainment	GDP per capita	All Components	Life Expectancy	Educational Attainment	GDP per capita	All Components	Adjustments to:	All Components
Range: Maximum	20	11	3	24	18	14	6	22		22
Minimum	-19	-12	-5	-22	-18	-12	-5	-24		-24
No. of Positive Changes	57	53	33	59	74	59	46	77		77
No. of Negative Changes	61	45	36	56	65	72	40	75		75
Distribution: -16 and below	1	0	0	5	2	0	0	4		4
-15 to -11	4	2	0	2	6	2	0	3		3
-10 to -6	11	14	0	10	19	7	0	15		15
-5 to -1	45	29	33	39	38	63	40	53		53
0	12	32	61	15	21	29	74	8		8
1 to 5	41	39	36	41	49	46	46	50		50
5 to 10	8	11	0	11	19	12	0	20		20
11 to 15	5	3	0	4	5	1	0	4		4
16 and above	3	0	0	3	1	0	0	3		3
Standard Deviation	5.96	4.45	1.31	6.75	5.83	3.94	1.31	6.57		6.57
Spearman correlation coefficient between observed and adjusted HDIs	0.987	0.993	0.999	0.984	0.992	0.996	0.9996	0.990		0.990

All HDI variables are expressed as one minus their scaled values. With respect to the 1990 HDI, GDPs per capita were converted to logarithms prior to scaling. GDPs per capita are purchasing power adjusted. All data are taken from UNDP (1990, 1991).

Table 7  
Summary Table: Observed versus Adjusted HDI Rankings

Ranking Changes	Random Error Range ( $\mu$ ): +/-5% <sup>a</sup> , +/-10% <sup>b</sup> , +/-15% <sup>c</sup>											
	1990 HDI					1991 HDI						
	Life Expectancy	Educational Attainment	GDP per capita	All Components	Life Expectancy	Educational Attainment	GDP per capita	All Components	Life Expectancy	Educational Attainment	GDP per capita	All Components
Range: Maximum	18	10	4	23	14	9	4	23	14	9	4	12
Minimum	-12	-10	-5	-16	-16	-7	-5	-16	-16	-7	-5	-25
No. of Positive Changes	51	58	39	57	76	62	47	57	76	62	47	78
No. of Negative Changes	64	49	35	75	71	75	43	75	71	75	43	74
Distribution: -16 and below	0	0	0	1	1	0	0	1	1	0	0	2
-15 to -11	1	0	0	2	3	0	0	2	3	0	0	4
-10 to -6	15	13	0	17	20	5	0	17	20	5	0	16
-5 to -1	48	36	35	43	47	70	43	43	47	70	43	52
0	15	23	56	13	13	23	70	13	13	23	70	8
1 to 5	33	48	39	36	55	54	47	36	55	54	47	49
5 to 10	14	10	0	11	19	8	0	11	19	8	0	24
11 to 15	2	0	0	5	2	0	0	5	2	0	0	5
16 and above	2	0	0	2	0	0	0	2	0	0	0	0
Standard Deviation	5.25	3.75	1.41	5.95	5.10	3.08	1.31	5.95	5.10	3.08	1.31	5.93
Spearman correlation coefficient between observed and adjusted HDIs	0.990	0.995	0.999	0.987	0.994	0.998	0.996	0.987	0.994	0.998	0.996	0.992

<sup>a</sup> <sup>b</sup> <sup>c</sup>: Random error range for high income, middle income and low income countries respectively. All HDI variables are expressed as one minus their scaled values. With respect to the 1990 HDI, GDPs per capita were converted to logarithms prior to scaling. GDPs per capita are purchasing power adjusted. All data are taken from UNDP (1990, 1991).

However, for the majority of countries, changes in rankings are relatively minor. If we observe the distribution of ranking changes for all countries included in our re-calculations of the HDI, 95 of the 130 countries in 1990 and 111 of the 160 countries in 1991 experience ranking changes of between -5 and +5 places only. Moreover, if we calculate rank correlation coefficients between the true HDI and its observed counterpart based on a measurement error in all components of the index within the range of -10 and +10 percent, we find coefficients of 0.984 and 0.990 in 1990 and 1991 respectively. Given the closeness of these coefficients to 1, we draw the conclusion that the HDI is quite robust to measurement error within this range.

HDI values were also adjusted for errors which are assumed to be a decreasing function of a country's income classification. Low income countries were assigned random errors in the range of -15 and +15 percent, while middle and high income countries were assigned ranges of -10 and +10 and -5 and +5 respectively.<sup>16</sup> As shown in Table 7, the greatest absolute change in rankings after adjustments to all components is 23 and 25 places for the 1990 and 1991 HDIs respectively, but again most countries experience ranking changes with the range of -5 and +5 places. The rank correlation coefficients between observed and true HDI values are 0.987 for 1990 and 0.992 for 1991. Once again, the HDI remains quite robust with respect to possible measurement error.

The results of a decomposition analysis of the sensitivity of the HDI to errors in each of its individual components are also reported in Tables 6 and 7. Judging from the range of ranking changes and the standard deviations of these changes, the HDI is most sensitive to errors in its life expectancy component. For the 1990 and 1991 HDIs, confining errors to this component saw absolute ranking changes of up to 18 and 16 places respectively. Confining measurement error to GDP per capita saw easily the smallest variation. Indeed, with the 1991 HDI version, imposing measurement errors within the range of -10 and +10 percent to this component resulted in no change to the rankings

of 70 countries. Of course, these results are a decreasing function of the variation of each of the components. For the 1990 HDI, coefficients of variation in life expectancy, educational attainment and GDP per capita are 0.168, 0.381 and 1.068 respectively, and for the 1991 index, 0.161, 0.386 and 1.020 respectively. Comparative variation aside, it is the general magnitude of variation of each of the HDI's component variables which lies at the very heart of its robustness with respect to measurement error.

Putting aside errors in GNP per capita, these results suggest that those reported in the preceding section hold after allowing for measurement error. This is made explicit in Table 8. Reflecting the high correlation coefficients between the observed and adjusted HDIs, the rank-order correlation coefficients between GNP per capita and the adjusted HDI for all country samples remain indicative of redundancy at either Level 1 or Level 2.

## VI POSSIBLE CHANGES TO THE INDEX

The results of the preceding sections show that the HDI is least redundant, and therefore makes its greatest contribution to assessment of inter-country development levels, when used to compare broadly similar groups of countries (i.e., grouped as high, middle and low human development). By contrast, if used to compare all countries it adds little new information to that provided by per capita income or any of the index's components. There are two responses to this. The first, which we favour, is to restrict HDI-based comparisons to country sub-samples. The second, which permeates the *Human Development Reports*, is to try to modify the index to make such comparisons more meaningful. Here we first discuss this latter course. We also suggest other modifications that might usefully be made to the HDI.

High correlation between the HDI's components in large samples is a difficult problem to overcome. As mentioned, for reasons of parsimony, the index could well be based on any one

Table 8

## Spearman Rank Correlation Coefficients between Adjusted HDI and GNP per capita

	Random Error Range ( $\nu$ ): +/- 10%				Random Error Range ( $\mu$ ): +/- 5% <sup>a</sup> , +/- 10% <sup>b</sup> , +/- 15% <sup>c</sup> .			
	1990 HDI		1991 HDI		1990 HDI		1991 HDI	
	GNP per capita & Adjusted HDI	Observed HDI & Adjusted HDI	GNP per capita & Adjusted HDI	Observed HDI & Adjusted HDI	GNP per capita & Adjusted HDI	Observed HDI & Adjusted HDI	GNP per capita & Adjusted HDI	Observed HDI & Corrected HDI
Low Human Development Countries	0.438**	0.723	0.628**	0.828	0.617**	0.828	0.693**	0.906
Medium Human Development Countries	0.304**	0.916	0.507**	0.900	0.342**	0.907	0.509**	0.889
High Human Development Countries	0.471**	0.978	0.524**	0.980	0.464**	0.957	0.520**	0.967
Developing Countries	0.616**	0.721	0.717**	0.844	0.693**	0.849	0.907*	0.987
Industrial Countries	0.770*	0.983	0.850*	0.988	0.786*	0.984	0.825*	0.941
All Countries	0.872*	0.984	0.904*	0.990	0.879*	0.987	0.850*	0.992

\*: Level 1 redundancy. \*\*: Level 2 redundancy. <sup>a</sup>, <sup>b</sup>, <sup>c</sup>: Random error range for high income, middle income and low income countries respectively. All HDI variables are expressed as one minus their scaled values, with the exception of the GDPs per capita used in the 1990 HDI. With respect to the 1990 HDI, GDPs per capita were converted to logarithms prior to scaling. GDPs per capita are purchasing power adjusted. All data are taken from UNDP (1990, 1991).

of its three components. However, in this event, the problem of redundancy *vis-a-vis* its components remains. That is, the index would remain redundant with respect to the excluded variables. Moreover, this option may be somewhat awkward from the UNDP's perspective given its apparent attachment and promotional capital invested in the index. An alternative solution is to modify the index using additional variables which satisfy the extremely demanding criterion of influencing the index to the extent that it is no longer redundant with respect to its components and GNP per capita. Indeed, UNDP in the 1991 *Report* does this by proposing distribution-adjusted and gender sensitive HDIs. Regrettably, however, due to data limitations, these indices can only be applied to a much narrower band of countries. For this option to become operational the issue, as Desai (1991), McGillivray (1991) and the UNDP itself acknowledge, becomes one of data collection. But there is no guarantee that the resulting index will provide any more insights than the recent versions. Indeed, the distribution-adjusted and gender-sensitive HDIs have rank correlation coefficients of 0.989 and 0.804 respectively with GNP per capita.

Two further courses of action could be pursued. The first concerns the redundancy between the HDI and GNP per capita. Given that of all components, GDP per capita is most highly correlated with GNP per capita, a weight of less than one-third (possibly even zero) could be applied to the former. This would be consistent with the general philosophy of the *Report* that income "is merely useful and is for the sake of something else" (Aristotle, quoted in UNDP 1990, p.9). However, implementing this is of little success. Even with a zero weight to GDP per capita, the rank correlation coefficient between the 1991 index and GNP per capita is 0.832 for the full country sample, and 0.813 and 0.882 for the developing and industrial country samples respectively. Against this background, a third option emerges: for the UNDP in future *Reports* to desist from its grandiose claims about the contribution of the HDI, especially in relation to GNP

per capita. Rather than seemingly seeking to legitimise and enhance the contribution of the HDI on these somewhat precarious grounds, it should perhaps confine itself to the more widely accepted, and indeed justifiable, philosophic basis of the index. More generally, as Pyatt (1991) implies, the UNDP should possibly desist from such grandiose claims irrespective of the context in which they are made.

There are two additional aspects of the HDI which could well benefit from refinement: the definition of the educational attainment variable and year-on-year comparability of the index. As mentioned in Section II, the 1990 *Report* used adult literacy as a measure of educational attainment, the 1992 *Report* uses a weighted average of adult literacy (two thirds) and mean years of schooling (one third). The weights are, of course, arbitrary and we have already concurred with Hopkins' (1991) view that not much is to be gained by suggesting alternative weighting procedures.<sup>2</sup> However, literacy and years of schooling are variables of very different orders of magnitude - the former being typically ten to twenty times the latter. To combine these two measures with weights of two-thirds and one-third (as the wording of the *Report* suggests is the objective) would require first indexing the variables (possibly so that the maximum value equals 100).

The HDI from different years is not comparable for two reasons. One is the redefinition of the education and income variables. The other is the range used to scale the variables (their desired and minimum observed levels) changes each year. (The ranks are not comparable for the further reasons that the 1991 *Report* contains 160 not 130 countries and that it ranks top-down, whereas the 1990 *Report* was bottom-up). The 1991 *Report* explicitly recognises this problem by producing a technical annex entitled "Measuring Progress in Human Development" (see pp. 96-97) which calculates the HDI for 110 countries for the years 1970 and 1985 on a consistent basis.

Whilst refinements in the HDI are desirable, so is



consistency and comparability across time. The rationale for shifting the range (a concept of relative poverty) does not, in our view the advantages to be gained from having a comparable time series. We would recommend that maxima and minima be set for each variable that may be used across a range of reports.<sup>17</sup>

## VII CONCLUSION

We have reviewed the usefulness of the UNDP's HDI from a number of perspectives. In terms of assessing development levels, we find the HDI to be largely redundant *vis-a-vis* GNP per capita at varying degrees for various country samples. Notwithstanding, it tends to make its largest contribution when looking at country sub-samples classified according to observed human development levels. As is the HDI itself, these conclusions are shown to be robust in the face of possible measurement error. Attempts to differentiate across the whole range have proved unsuccessful and we suspect this will remain the case and would suggest the index be given a more limited application than some of the grandiose claims in the *Human Development Reports*. We also make a number of suggestions on improving the index, regarding weights, composition and comparability.



## NOTES

1. Such appraisals can be found in Desai (1991), Rao (1991) and Hopkins (1991).
2. It ought to be emphasised that our comments, for the moment, are confined to what the UNDP in its 1991 *Report* now calls its "basic" HDI only, and therefore ignore innovations such as the gender-sensitive, distribution-adjusted and inter-temporal HDIs (UNDP, 1991, Technical Notes 3 to 5).
3. It should also be noted that in the 1991 *Report*, countries were ranked in ascending HDI order, while in 1990 they were ranked in descending order. Table 1 above ranks countries in both years in descending order for comparative reasons only.
4. We have used the purchasing power parity (PPP) GDP per capita for this Figure. The HDR uses GNP per capita - which increases the disparity. We prefer the former measure as both the use of domestic, rather than national, product and applying PPP exchange rates give a more accurate income-based measure of development.
5. The population figures used were for 1988, since 1987 was not provided in the Report. A country's "total HDI" was calculated in the same way as for GDP, i.e. the product of HDI and population.
6. The gini of world income distribution of 0.53 is lower than values often cited (in the range 0.65 to 0.70) as PPP GDP has been used for the calculations - as is well known this decreases income disparities between countries.
7. Figures 7 and 8 are drawn using the actual country data and so are not strictly comparable. The main point, that both treatments, give a more or less horizontal line above the cut-off is however quite clear.
8. Such a high coefficient is also not surprising since there are over one hundred countries below the threshold, and their ranks change very little (only the denominator of their income term changes). On the other hand, for those countries whose income is more than three times the threshold the Spearman's is only 0.74. This rises to 0.95 for those countries whose per capita GDP is two to three times the threshold.
9. Many of these differences are in excess of 20 ranks and even as high as 56.
10. For further details of the PQLI, see Morris (1979).
11. For further details of this index, see McGranahan *et. al.* (1972).
12. Larson and Wilford, using a sample of 150 countries, concluded that the PQLI was redundant *vis-a-vis* GNP per capita on the basis of a rank correlation coefficient of 0.776. McGillivray drew the same conclusion concerning the 1990 HDI and GNP per capita for a sample of 119 countries on the basis of a rank correlation coefficient of 0.889.

13. The  $R^2$  from a regression of ranks is the square of Spearman's coefficient; 0.71 is the square root of 0.50.

14. All errors were generated using the @RAND command in Lotus 1-2-3 Release 2.3. This command generates errors which follow a triangular distribution.

15. The exception to this being countries whose populations were recorded as being below 1 million in the 1990 *Report*.

16. Income classifications were obtained from UNDP (1990, 1991).

17. For adult literacy the maximum of 100% presents itself. For the other variables maxima should be set above the conceivably attainable values over the next, say, ten years (e.g. 85 for life expectancy). This will mean that no country may score the maximum of unity, but this seems no particular loss. The setting of minima is also problematic - if the lowest current value is used a country may fall below that in a subsequent period. One alternative is to set minima of zero, the other is to pick values below which it is believed no country will fall.

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