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Knibbs: a pilgrim of a new world in demographic theory

A chapter from a PhD Thesis: entitled 'Considerations for a two-sex demography: when, why and how should both sexes matter to demography?

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Canberra, April 1996

Abstract

This paper corresponds to Chapter 7 in the Ph.D thesis entitled 'Considerations for two-sex demography: when, why and how should both sexes matter to demography?'. The thesis comprises two mains parts, Part I - 'Where was the beginning? What was it?' - and Part II - 'When, why and how should both sexes matter to demography?'. Chapter 7 closes Part I, which places the two-sex demography in the wider context of the development of demographic theory since its birth and earlier growth.

The six chapters comprising Part I search for the most important anticipations of a two-sex demography. First, the sex ratio is shown to have been not only the first demographic measure ever created in demography but also the simplest and perhaps most important two-sex concept. Chapter 2-4 discuss the sex ratio as a 'matter of fact', in association with Graunt's of 1662, and as an 'explanatory resource', in association with Arbuthnot's (1710) first test of significance of a statistical hypothesis. Secondly, the 'passion between the sexes' was first used by Malthus to highlight the role of sexual reproduction in demographic change as part of his dual principle of population (Chapter 5). In Chapter 6, a fascinating overview and periodization of the evolution of the demographic concept of fertility is explored. Chapter 6 is the pivotal chapter of the thesis because provides the grounds for the four chapters comprising Part II. In particular, it shows that there is a need to acknowledge the three meanings embodied by the demographic concept of fertility: fecundity, fertility output, and fertility outcome; curiously, these three conceptual meanings seems to have secured recognition in demography over a long historical process separated by about hundred years each. Moreover, Chapter 6 maintains that the understanding of these three scientific breakthroughs in the evolution of the demographic concept of fertility are important to understand what can be termed the necessary and sufficient conditions for a comprehensive two-sex demography.

Chapter 7 closes Part I of the thesis with the most important anticipation of a two-sex demography found in Knibbs's work, *The Mathematical Theory of Population* has published in 1917. Knibbs's massive work of 1917 continues to be generally seen as an unusual document. In part, this is because what was supposed to be just an Appendix to the 1911 Australian Census turned out into a sophisticated, elegant and comprehensive essay of 466 pages. Another reason is that Knibbs's book is perceived to be ahead of its time, mainly because its emergence independently of Lotka’s stable population theory.

Yet, Chapter 7 shows that certainly more unusual is the slight attention given to Knibbs’s *Mathematical Theory of Population* in conventional demography, even in situations where this work should be a compulsory reference. Some striking examples of the neglect of Knibbs's work are provided, namely in field such as fertility, nuptiality, and the so-called 'two-sex problem'. Then, the paper shows that the *Mathematical Theory of Population* provides the first two-sex theory of the probability of marriages in age groups can be found. Following Quételet and Körösi (discussed in Chapter 6), Knibbs considered more fully and completely the possibility and feasibility of taking into consideration the role of both sexes in studies of population. Although Knibbs did not advance much into a conceptualization of the fertility in the way outlined in the thesis, his theory of population clearly anticipates at least the necessary condition for a two-sex demography: that for certain purposes the methodological frameworks should explicitly take into consideration the numbers and behaviour of both males and females. Moreover, Knibbs's invention of the concept of 'marriage function' can be seen as an ingenious way of resuming the centrality of marriage so cherished by authors like Graunt and Malthus. Indeed, the notion of marriage and mating functions seem to be indispensable for the transformation of relatively abstract concepts such as the 'passion between the sexes' into operational measures of the interaction between the sexes. Seen as a function, marriage becomes the object of modelling, a development that some three decades after Knibbs's work in the 1910s would lead to the two-sex endeavour best known as the two-sex problem.

> An ideal theory of population is one which would enable the statistician not only to determine definitely the influences thereupon of the various elements of human development, and of the phenomena of Nature, but also to examine all facts of interest to mankind, as they stand in relation to population. And however hopeless may be the expectations of establishing such a theory with meticulous precision and all detail, it nevertheless remains true that fluctuations of population can often be adequately understood only when they are analysed by means of definite mathematical conceptions (Knibbs, 1917: 3).

The cradle of the two-sex demography: why Australia?

In conventional demographic analyses most of the role played by the principle of complementarity between the sexes is generally taken for granted, or even deliberately ignored. This is not because demographers are unaware of complementarity in demographic phenomena; a great deal of demography stands on principles which are exactly the opposite of complementarity. For some purposes, for instance, a neuter approach which abstracts even from the standard variables age and sex may be enough; for others there is a need to strip off a layer of appearances which are misleading and which impede the study of demographic reality. As Keyfitz pointed out, in a paper published in 1980, there are several cases in which one can easily draw 'a wrong conclusion from exact statistical data and even when they are known to be quite accurate' (Keyfitz, 1980: 48). A large array of concepts, measures and models which have led to the one-sex demography have been developed with the objective of digging into the depths of demographic relations which strongly influence what appears on the surface. 'Indeed', Keyfitz (1980: 63) remarked further, 'perhaps the biggest difference between professional demographers and others who deal with population is that the professionals know just enough to realise that the surface phenomena are influenced by these deeper ones'.

This explains, at least in part, why demographers of the twentieth century have been much more concerned with the separation than the complementarity between the sexes. However, the separation of the sexes and the method of controlling or stripping off the effect of, say, population structure, have been very effective for some purposes but not for others. Some aspects of the deeper layers of demographic phenomena need to be studied through methods which combine and integrate the role of both sexes. Demographers are aware of this, including those who have simply attempted to adapt mechanically their one-sex methods to the demands imposed by phenomena which can only be understood through the mechanisms of complementarity between the sexes; because they have tried to modify the one-sex methods in an *ad hoc* manner, in general their theoretical frameworks have become creaking and ugly edifices.

In the context of the alternatives between one-sex and two-sex approaches the sex ratio plays a sort of arbitration role that has proved to be paramount for the consistency of any demographic model. Although, if not because, the sex ratio is two-sex by its nature and the most simple composition measure available in demography, during the twentieth century it has already been much more used in one-sex than two-sex demographic approaches.

Historically, the use of the sex ratio in the twentieth-century demography can be traced to two important directions, both sketched and made public in the 1910s. These two directions were developed independently of one another, and while one applied the sex ratio in the construction of a one-sex approach the other applied it in the construction of a two-sex approach. The former emerged in Europe and the United States, when Böckh created the net reproduction rate and more fully when Lotka and his co-authors developed the mathematical model of classical stable population theory (see Part II).

On the other hand, in Australia Knibbs seems to have been totally busy for a great part of the 1910s with his massive work, *The Mathematical Theory of Population, of Its Character and Fluctuations and of the Factors which Influence Them.* This work, first published in 1917, was written as an Appendix to the 1911 Australian Census, though its sophistication and depth surpassed any expectation for an appendix, to the extent that even contemporary authors still refer to it as a 'highly unusual document' (Gray, 1988: 5).

The reason Knibbs's *Mathematical Theory of Population* has been considered an unusual document is because of its comprehensiveness and, perhaps, even more relevant, because of its emergence in parallel and independently of Lotka's stable population theory.

> *The Mathematical Theory* was a highly unusual document. It was certainly ahead of its time, as Wilson contends, but this was in part because it was hardly possible to attempt an undertaking of its type in the second decade of the twentieth century, before the development of stable population theory. The theoretical emphasis of the work is the search for immutable mathematical laws which describe the components of population structure and growth, laws which ultimately could not be justified. On the other hand, the book contains a large number of ideas for statistical methods and measures, especially in fertility and mortality. Some of these ideas have become standard methods of demographic analysis in the second half of the twentieth century, one suspects in most cases after rediscovery. Other ideas remain to be recycled into use (Gray, 1988: 5-6). [1]

Also unusual is the slight attention given to Knibbs's *Mathematical Theory of Population* in conventional demography, even in situations where this work should be a compulsory reference. For instance, the huge and comprehensive 'inventory and appraisal' edited by Hauser and Duncan in 1959 made no single reference to Knibbs's *Mathematical Theory of Population*; even Lorimer (1959), in his otherwise very interesting overview of the development of demography, found no reason to mention Knibbs's original demographic work. This neglect has not been overcome in the last three decades or so; for this reason, statements such as the following from Caldwell are not just very rare but probably dismissed as exaggeration: 'The modern attempt to examine global population and rates of change

[1] Gray wrote this paper to question Wilson's claim in 1986 that Knibbs was not the real author of the work *The Mathematical Theory of Population*. In an address to mark the fiftieth anniversary of his appointment as Commonwealth Statistician, Sir Roland Wilson referred to the first two Commonwealth Statisticians Sir George Knibbs and Mr Charles Henry Wickens and maintained that the latter was the one who wrote that book (Gray, 1988).

originates in Australia with George Knibbs's (1917) remarkable Appendix to the 1911 Census' (Caldwell, 1985a: 23).

If Caldwell's remark about Knibbs's contribution to modern demography is not an exaggeration, it is reason to wonder: 'How can it be explained that a central figure in twentieth century demography has been so excluded from contemporary reviews of the history of modern demography?'. Two cases are particularly striking, one related to fertility and the other to nuptiality.

There is a reason why I have singled out the name of Lorimer from many other authors who have dealt with the history of modern demography. Although in his 1959 review Lorimer failed to acknowledge the originality of Knibbs's demographic work, he provided an interesting distinction as to the tradition of fertility conceptualization in Europe and United States:

> Fertility is traditionally conceptualized in Europe as 'fertility of marriages' and in the United States as 'fertility of persons' (by sex and age) - marital status being treated merely as one of the conditions influencing reproductive behavior. This difference is probably due in large part to differences in types of available data. It may also be due in part to the influence of scientists with biological orientation, notably Pearl and Lotka, on American demography in the 1920's. But the difference in approach also reflects differences in real situations ... In any case, European demographers have tended to place greater emphasis than their American colleagues on the differentiation between the formation of conjugal unions and nuptial fertility as major components in total fertility (Lorimer, 1959: 143).

Knibbs's conceptualization of fertility could not be accommodated in Lorimer's characterization of the existing traditions, not because Knibbs was neither European nor American, but because his *Mathematical Theory of Population* conceptualized fertility as 'fertility of marriages' as much 'fertility of persons'. On these grounds, Knibbs's approach was at odds with tradition and, perhaps, too much ahead of its time to even deserve a reference.

But the neglect of Knibbs's work does not stop here. Another field in which Knibbs has generally been missed out is in the so-called 'two-sex problem' and, in particular, the consideration of nuptiality from a two-sex point of view. In conventional literature, the formal treatment of the 'problem of the sexes' is generally traced to the work of the French demographer Vincent (1946), and that of the two Australians, Karmel (1947, 1948a, b, c) and Pollard (1948). A few notorious examples where Knibbs's treatment of the problem of the sexes has been completely ignored are the works of Pollard (1973), Pollak (1990), Schoen (1988), and Smith and Keyfitz (1977).[2]

With regard to the conceptualization of fertility, Knibbs's *Mathematical Theory of Population* has been ignored perhaps because it was on the margin of the two main traditions

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[2] Feeney has been one of the few authors, if not the only one, among those interested in the 'two-sex problem' to acknowledge Knibbs's authorship of the concept of 'marriage function'. Feeney did this in his 1972 Ph.D Dissertation 'Marriage rates and population growth: the two-sex problem in demography', though he only paid attention to Knibbs's definition of the concept of 'marriage function' on page 214 of *The Mathematical Theory of Population*. Feeney's main goal in his dissertation was 'to systematically explore the structure of the class of all mathematical functions which may express the dependence of numbers of marriages in a population on the numbers of males and females available for marriage' (Feeney, 1972: 15).

of conceptualizing demographic reproduction; a similar explanation can be found for the nuptiality and the 'two-sex problem'. Nuptiality has been conceptualized during the twentieth century in association with the female population and only seldom in terms of combination and interaction of both sexes. Moreover, with regard to the interaction between the sexes Knibbs saw the subject not just as a formal or mathematical issue but as an empirical problem treated on an equal footing. In contrast, in the 'two-sex problem' the subject has been treated predominantly as a mathematical problem and as a reaction to the one-sex nature of Lotka's stable population theory; in this case, the empirical has been used almost exclusively as an illustration and subsidiary of the formal models, but not a matter valid on its own.

In 1947 and 1948, Karmel and Pollard proposed the first two mathematical models intended to reconcile the male and female net reproduction rates in stable and non-stable populations. Ever since, the aspiration to replace the one-sex nature of stable population theory has provided motivation for unprecedented growth of the research on the two-sex problem in formal demography. However, the interaction between the sexes cannot be ascribed to the stable population theory, nor even to a mathematical difficulty only. This has been demonstrated 1980s and 1990s by some of the treatment of the interaction between the sexes from a two-sex perspective in broader terms (Schoen, 1988, 1993; Pollard and Höhn, 1994). The question of the sexes related with stable population theory is just one part of the broad scope of the relevance of a two-sex approach in studies of fertility and nuptiality. This is corroborated, for instance, by Schoen's (1988: 121) definition of the 'two-sex problem' as '... the inability of conventional population models to capture the changes in nuptiality and fertility rates that are produced by changes in population composition', even though Schoen still remained prisoner of the misconception which traces the origin of two-sex demographic research to the work of Karmel (1948c) and A. Pollard (1948) only.

This chapter is particularly concerned with creating the basis to relieve the debate on the interaction between the sexes, an important aspect of the principle of demographic complementarity, from the two misconceptions identified above. There are two advantages in doing this while reviewing Knibbs's work. On the one hand, this review is intended to challenge the neglect of Knibbs's work in the development of demographic theory during the twentieth century. On the other hand, the 'fertility of persons' and 'fertility of marriages' become parts of the issue from a two-sex point of view and Knibbs's book illustrates this rather convincingly. After all, from my review of the anticipations of the two-sex demography, I have found no other work like Knibbs's *Mathematical Theory of Population* which provides a balanced, though brief, a framework for a two-sex approach in the theoretical, formal and empirical areas of demography.

Knibbs's attempt to bring together the two most important traditions in the conceptualization of fertility, those identified by Lorimer in 1959, lay dormant for about three decades. There is no doubt that Karmel and Pollard were the authors who set the new research agenda in the late 1940s, but they did it more by resuming and placing the problems in investigation in a new context than starting from scratch. Karmel, in his PhD thesis, showed that he was well aware of Knibbs's and Körösi's work. The works of Karmel and Pollard, both Australian authors, have led to a new field in formal demography; this seems enough to point to Australia as the cradle of two-sex demography. This does not mean that the contributions of authors from elsewhere are of less importance, but if one can already speak of a certain tradition in the demographic conceptualization of fertility and nuptiality from a two-sex point of view, Australia has certainly been the source of the first and most important initiatives.

On the question, 'why Australia?', perhaps Lorimer's own classification provides a plausible explanation. In a way, while Australian demographers seem to have often hesitated between the two main demographic traditions developed in Europe and United States, Knibbs can be credited as the pioneer of truly new world in demographic theory. Quételet had perhaps envisaged it, and Körösi made an original and elegant research in term of a one-sex and two-sex approaches of fertility. But Knibbs treated the subject in a comprehensive way, linking the theoretical and formal, both mathematical and geometrical, as well as the empirical. Moreover, the fact that later the precedent created by Knibbs was followed by another two Australians may reflect differences in real situations from those that Lorimer found in Europe and United States. After all, the twentieth century European and American demographers have grown within their own strong traditions. The Australian demographers, in turn, had to approach both traditions from elsewhere, and certainly they found a way to claim a tradition of their own which had never been explored before.

I will review the eve and background of two-sex demography by placing attention on Knibbs's *Mathematical Theory of Population* and other works of his. The objective of this chapter is to demonstrate that it is possible to speak of a two-sex approach tradition set by the attempts to conceptualize fertility not just as fertility of marriages or fertility of persons in disregard of the methodological approach behind them. As Knibbs indicated, the one-sex approach used to deal with fertility and nuptiality needs to be placed in the wider context provided by a two-sex approach.

Population in the aggregate: sex ratio, multiple births and human reproduction

Knibbs's *Mathematical Theory of Population* comprises a total of 466 pages divided into eighteen chapters. The first eight chapters introduce several issues on the theory of population, such as: the nature of demographic problems; the necessity for the mathematical expression of the conditions of demographic problems; various types of population fluctuations; group values, their adjustment and analysis; ways of summation and integration for statistical aggregates; the place of graphics and smoothing in the analysis of population-statistics; and conspectus of population-characters (Knibbs, 1917: 1-107). Chapter 9 focuses on population as an aggregate, including its distribution by sex and age, while Chapter 10 discusses the 'masculinity of population'.

Today Knibbs would most probably not dare to use the terms masculinity and femininity in the dispassionate and technical fashion he did; but in his time demographers could not envisage that some decades later such concepts would be considered guilty of androcentric stereotypes. Without using the term ‘sex ratio’, Knibbs discussed its content through its two well known surrogates, ‘masculinity’ and ‘femininity’, and covered the following issues: the norms and the various definitions of masculinity and femininity; the use of norms for persons and masculinity only; the relation between masculinity at birth and general masculinity of population; masculinity of still and live nuptial and ex-nuptial births; coefficients of ex-nuptial and still-birth masculinity; masculinity of first-born; masculinity of populations according to age, and its secular fluctuations; and theories of masculinity (Knibbs, 1917: 130-141).[3]

Knibbs returned to the significance of the sex ratio later, in two papers published in 1925:

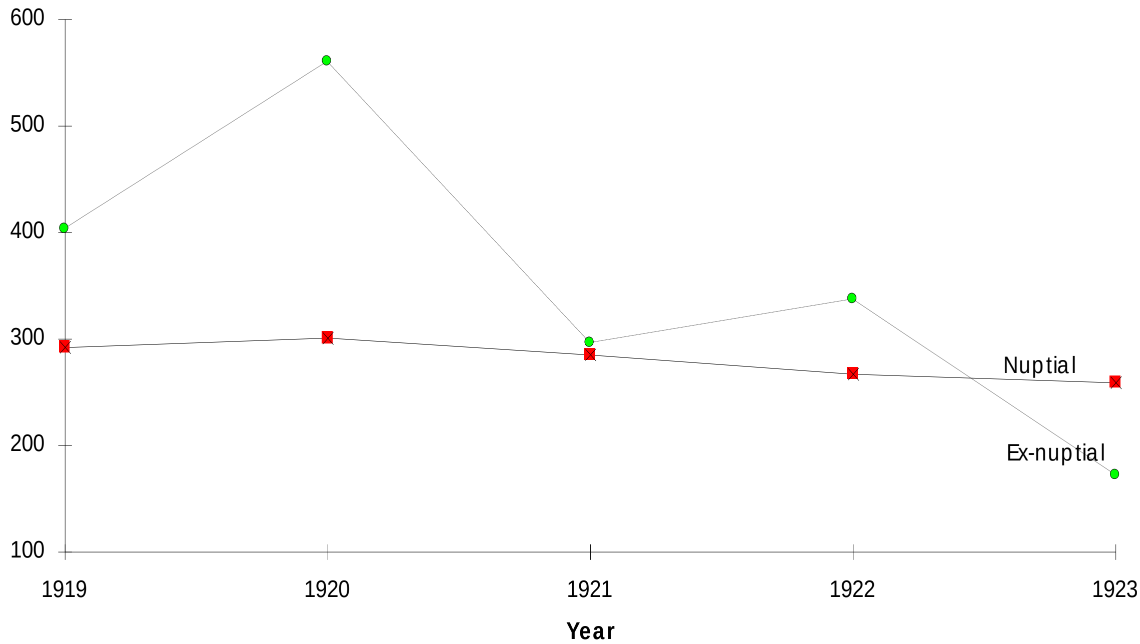
> The phenomena of the sex-ratios of various forms of life are of the first order of importance, and among them, those which throw light upon the make-up of human population are of special interest (Knibbs, 1925a: 212).

Contrary to 'our predecessors', as Westergaard (1932: 72) put it, Knibbs was not just struck by the regularity of the sex ratio, and paid equal attention to its deviations. ' "Masculinity" may be expressed ...', so Knibbs defined his most used surrogate of the sex ratio, 'by the difference between males and females divided by their sum; that is (M-F)/(M+F)’ (Knibbs, 1925a: 213). Figure 1.7.1 depicts graphically the data provided in Knibbs's 1925 articles: 'Per 10,000 nuptial births and per 10,000 ex-nuptial births in Australia from 1919 to 1923 the masculinities were respectively as follow' (Knibbs, 1925a: 213).

[3] Following his finding that masculinity of still-births was considerably higher than that of live-births, and that masculinity at birth was about 1.05 or 1.06, Knibbs remarked about the various attempts to explain the masculinity at birth:

> J. A. Thomson is his ‘Heredity’ says that, according to Blumenbach, Drelincourt in the 18th century brought together 262 groundless hypotheses as to the determination of sex, and that Blumenbach regarded Drelincourt’s theory as being the 263rd. Blumenbach postulated a ‘Bildungstrieb’ (formative impulse), but this was regarded as equally groundless. It has been suggested that war, cholera, epidemics, famine, etc., are followed by increase in the masculinity. These will have to form the subject of later investigations. At present it would seem that the first necessity is a sufficiently large accumulation of accurate statistics, as a basis for study. The one point which is clear is that death *in utero* (at least in the later stages) is marked by much greater masculinity than that which characterises live-births. This will be referred to later in dealing with infantile mortality (Knibbs, 1917: 140-141).

Figure 1.7.1 Australian masculinities at bith, 1919-1923



Source: Knibbs, 1925a: 213

'The irregularity of the ex-nuptial case is very striking', Knibbs added about the data and called attention to the 'mean square deviation' in the final column of his table: 15.6 for nuptial and 127.8 for ex-nuptial. Furthermore, he compared the ratio of males to females for still and live-births in several European countries and found that the former was about 1.305, the latter was about 1.070, and the ratio between the two was 1.220. These findings confirmed those Knibbs had presented in the *Mathematical Theory of Population* for the Western Australian population, and this led him to conclude:

> That this sex-ratio, males to females, is invariably greater for still than for live-births indicates that male lives are in greater jeopardy prior to birth than are female lives (Knibbs, 1925a: 214).

With regard to the effect of multiple births on masculinity Knibbs wrote in the same article:

> I have shown elsewhere that ratio of males to females is reduced by multiple births ... Inasmuch as everywhere the numbers of multiple births are relatively very small (for example, in Australia per ten million confinements there are only 98,020 twins, 829 triples, 15 quadruplets, and perhaps 2 quintuplets, or, roughly, 1 in a hundred confinements for twins, and about 1 in 10,000 for triples), it is evident that these can affect the general masculinity but very slightly ... For this reason the variation of masculinity with size of family, which will be shown hereinafter to occur, must be regarded as a fundamental fact in the phenomena of human reproduction, just as much so as the production of fertile male ova exceeds in number that of fertile female ova, and not regarded merely as a consequence of multiple births (Knibbs, 1925a: 215-216).

By drawing attention to specific empirical data from different sources and some possible mathematical equations Knibbs sought to outline a general law of diminution of masculinity with increase of family . He found no simple curve which could represent exactly the results, but on the grounds of the empirical evidence concluded that: first, on the average large families tended to have more females than small families; this result seemed to be more defined for families of 1 to 6, after which possibly the masculinity was less well marked. Secondly, multiple births markedly confirmed previous findings, but because they were relatively small in numbers, they quantitatively affected the general result but slightly. Thirdly, a more extensive study was needed and should embrace separately the living issue, the deceased issue, and both combined; as yet, he found it important to compute the results from male-parent records and female-parent records separately, as well as in combination. Fourth, Knibbs conjectured that a definite law could be expected to appear only when very large numbers of cases were studied. So, the possibility of a secular change with time and any improved knowledge of the phenomena of sex ratios in human reproduction depended on a systematic study carried out on more extensive scale.

The review so far has highlighted Knibbs’s applications of the sex ratio as a measure of matters of fact. In later papers he returned to the topic of masculinity of first births (1927a), multiple births (1927b) and a 'proof of the laws of twin-births' (Knibbs, 1927c). The next section stresses Knibbs's use of the sex ratio as an algorithm process or explanatory resource in sketching his theory of nuptiality and fertility.

From natality to fertility through nuptiality

Before turning to the core of Knibbs’s theory of the probability of marriages according to pairs of ages it is useful to give some attention to he conceptualized demographic analysis in general. Following some broad considerations on population in the aggregate, Chapter 11 of *The Mathematical Theory of Population* concentrates on 'Natality'.

> The phenomena of human reproduction, as affecting population, and the whole system of relations involved therein, may be subsumed under the term 'natality'. In one aspect they measure the reproductive effort of a population; in another they disclose the rate at which losses by death are made good; in a third they focus attention upon social phenomena of high importance (*e.g*., nuptial and ex-nuptial natality); in yet another they bring to light the *mode* of the reproductive effort (*e.g*., the varying of fecundity with age, the fluctuation of the frequency of multiple-birth, etc.) (Knibbs, 1917: 142).

Following this broad definition of the scope of natality, Knibbs detailed each of the three features associated with natality in three separated sections. First, the study of birth-rates as part of natality in its narrow sense and in association with the Malthusian law concerning the arithmetical increase of food production as opposed to the geometrical increase of population(Chapter 11); secondly, the role of ‘Nuptiality’ (Chapter 12); and only then, in third place, do two chapters focus on fertility strictly speaking: Chapters 13 on ‘Fertility and fecundity and reproductive efficiency’ and Chapter 14 on ‘Complex elements of fertility and fecundity’.

Clearly, this conceptualization of demographic analysis and, especially the way fertility is placed in Knibbs's conceptualization of demographic reproduction, contrasts with most contemporary textbooks in demographic teaching. In particular, it contrasts with the view that nuptiality is not in itself of particular interest to demographers. According to Newell,

> Marriage, separation, divorce, widowhood and remarriage, collectively called 'nuptiality' in demography, are not in themselves of particular interest to demographers. Rather, their importance arises partly from their relationship with the age at which sexual relations begin and end, and partly with the formation and dissolution of families and households (Newell, 1988: 90).

This statement is completely at odds with Knibbs's position concerning the role of nuptiality in demographic analysis:

> The phenomena of reproduction have a double aspect, viz., one a sociological and the other a physiological. Thus, from the standpoint of a theory of population, both are important. The women of reproductive age in any community furnish the potential element of reproduction; but the resolution into fact depends also upon social facts as well as upon physiological; for example, the relative proportion of married and single, i.e., the nuptial-ratio, even more profoundly affect the result than physiological variations of fecundity (Knibbs, 1917: 175).

Knibbs started by conceiving natality in the context of the whole demographic system and in the standard and neuter perspective used in demographic teaching nowadays. Yet, contrary to the mainstream approach even before attempting to control and strip off the effect of population structure by focusing, for instance, on one-sex models and measures Knibbs considered some fundamental mechanisms in the interaction between the sexes. So, contrary to the widely accepted view, as it is depicted by Newell' statement quoted above, Knibbs addressed the demographic reproduction moving from natality to fertility not directly but through nuptiality:

$$\left\langle \begin{matrix} \text{DEMOGRAPHIC} \\ \text{REPRODUCTION} \end{matrix} \right\rangle \rightarrow \langle \text{Natality} \rangle \rightarrow \langle \text{Nuptiality} \rangle \Rightarrow \langle \text{Fertility} \rangle$$

A sketch of a two-sex approach on nuptiality: theoretical, formal and empirical

If the authorship of the last quotation from Knibbs's 1917 work were not known, one could well imagine it to have come from any of the contemporary authors who, in recent years, have admonished demographers to admit that social factors may more profoundly affect population change than physiological factors.

Although Knibbs's sketch of his analytical framework relevant to a two-sex approach is brief, fundamentally it is consistent with the principle of complementarity between the sexes discussed here. Moreover, Knibbs dealt with the interaction between the sexes and the age combination with elegance in its threefold dimension: theoretical, formal and empirical. Theoretically, Knibbs formulated and addressed key issues and concepts relevant to nuptiality analysis. Formally, Knibbs dealt with demographic theory and techniques of population using not just algebra and calculus but also geometrical and graphical representations.[4] Empirically, Knibbs applied population theory and technique especially to the data from the 1911 Australian Census.

Knibbs's *Mathematical Theory of Population* resumed and expanded, in an unprecedented way, the centrality of marriage and couples so cherished by earlier demographers such as Graunt and Malthus. Both in his main work of 1917 and several papers published in the 1920s, Knibbs revealed an explicit interest in the dual nature of demographic reproduction, namely the social and the physiological.

> As at present constituted, the social organism is the theatre of a conflict between controls and traditions (which are generally supposed to be of great social interest and value) and the gonad urges of the individual human organisms. Biological facts, which throw any light upon the features and trends of this conflict, have been at all times of scientific importance. Owing to the advance of knowledge in respect to the functioning of the endocrine and sex glands, and in respect to the technique of the control of their unrestricted play, the analysis of facts which reveal the features and drift of this conflict has become, quite recently, of very special importance. And certain aspects of this are accentuated in significance by existing and threatening difficulties arising from population-growth. These difficulties are world-wide (Knibbs, 1927a: 73-74).

This statement suggests that demographers' reliance on biological determinism may have been much more recent than it seems at first. Before the scientific discoveries, such as those of Darwin, there was little basis for a population approach based on biological determinism. Many interpretations of demographic phenomena were attributed to mystic or providential interventions. In addition, Knibbs's treatment of the subject of nuptiality before fertility indicates that he gave a privileged place to the role of social reproductive mechanisms in the overall process of demographic reproduction:

> The nuptial-ratio in any community may be regarded as a measure of the social instinct, and also a measure of the reproductive instinct, modified by social traditions as well as facilitated or hindered by economic conditions. This ratio, for the case of females, is, of course, specially important in relation to fecundity (Knibbs, 1917: 175).

The concept of 'nuptial-ratio' corresponds, in current terminology, to the measure of 'general marriage rate' (GMR). Knibbs discussed this measure while he identified the limitations of the crude marriage rate, particular its 'uncertain significance' as a measure, due to the fact that it is insensitive to the lack of homogeneity in populations. 'The heterogeneity', Knibbs (1917: 176) explained, 'arises largely from divergences of social life and tradition, in respect of the relative frequency of marriage, and the frequency according to age'.

Chapter 12, entitled 'Nuptiality', presents a systematic analysis on four major issues: it starts by providing some considerations on the concept of nuptiality and its specific

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4 'In general', Knibbs wrote about graphs of data and smoothing,

> we are concerned with two kinds of alteration; one may be called the '*redistribution of the data without alteration of their aggregate*;' and the other may be called the '*alteration of data to coincide with what is deemed the most probable result,*' having regard to all the facts (Knibbs, 1917: 85).

operational definitions; these are followed by a discussion on marital status and composition of population, what Knibbs called ‘conjugal constitution of the population’.

> The significance of marriage in respect of reproductive activity depends upon the relative frequency of nuptial and ex-nuptial births, as well as upon the relative proportions of the married and unmarried (Knibbs, 1917: 175).

In other words, it depends not merely upon the nuptial ratio, but also upon nuptial and ex-nuptial fertility, particularly during the reproductive period of life.

Secondly, Knibbs considered the norm of conjugal relations, especially divorce. On the latter issue, Knibbs discussed the secular increase of divorce, the abnormality of the divorce curve, and the desirable form of divorce statistics in order ‘to be of high value from the standpoint of sociology’ (Knibbs, 1917: 189).

> The frequency of divorce is of sociological interest. The effect of Divorce Act (55 Vict., No. 37) of New South Wales, and of Victoria (53 Vict., No. 1056), which came into force on 6th August, 1892, and 13th May, 1890, respectively, have had a conspicuous influence in increasing its frequency (Knibbs, 1917: 186).

Knibbs added that the sociological value of statistical data required the data to be classified at least according to age *per se*, to difference of age and to duration of marriage. These three aspects should make it possible to ‘expose the conditions which are of danger from the standpoint of social stability’ (Knibbs, 1917: 189).

Thirdly, Knibbs addressed the question of the interaction between the sexes under the title ‘Frequency of marriages according to pairs of ages’. Finally, he outlined briefly his general theory of protogamic and gamic surfaces. These two last issues deserve to be reviewed at some length here. On the one hand, they have been widely neglected in the demographic literature on nuptiality, including in the debates on the ‘two-sex problem’; on the other hand, they provide a broad and consistent background to Part II on the principles on which conventional demography has stood so far as compared with the aftermath of the complementarity between the sexes.

Nuptiality according to pairs of ages: the first two-sex mathematical model

In the section entitled ‘Frequency of marriages according to pairs of ages’ Knibbs (1917: 189-201) moved to the heart of the complementarity between the sexes. He started by saying that ‘The frequency of marriage according to pairs of ages can be well determined only for a considerable number of instances’ (Knibbs, 1917: 189). After illustrating this point with some examples, Knibbs provided a table for single year groups of number of marriages arranged according to the ages of the contracting parties and based on Australian data for 1907-1914; the data were drawn from the 1911 Australian Census and provided the empirical grounds for a detailed debate on nuptiality in Chapter 12. As part of this debate, Knibbs pointed out the various irregularities in the data, and discussed the errors in the ages at marriage and the ways of correcting such errors. He then considered the 'Probability of marriage of bride or bridegroom of a given age, to a bridegroom or bride of any (unspecified) age':

> The correction of the data, as indicated in the preceding section, admits of the construction of a table shewing in say 100,000 marriages the number occurring for bridegrooms of any given ages, and for brides of any given ages, the age of the other partner to the union being unspecified (Knibbs, 1917: 198).

Yet, as Knibbs pointed out a few pages below, grouping the data according to age-groups for single years

> is by no means perfectly satisfactory for the purpose of very accurately determining the frequency of conjugal-groups according to various *differences of age*. It is obvious that when all bridegrooms, whose age was say *x* last birthday, and brides whose age was say *y* last birthday (*x* and *y* being integers), are grouped, the group contains brides who are one-half year older than the difference *x-y*, as well as brides one-half year younger than this difference (Knibbs, 1917: 192).

So Knibbs considered the most possible and satisfactory way 'To properly determine the law of nuptial frequency according to specified differences of age'; after determining the marriage rates for the Australian population in the period 1907-1914 he represented 'the probability of a marriage occurring in a population of males, females, or persons' (Knibbs, 1917: 193). Because the number of 300,000 marriages was not sufficient for the determination of adequate data for single years, particularly at the higher ages, Knibbs tabulated the data by 5-year groups (uncorrected data). These data are reproduced in Table 1.7.1.

Table 1.7.1 Number of marriages arranged according to age at marriage in five year groups. Australia, 1907-14

Bride-groom's age	Brides' age 10-14	15-19!	20-24!	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65-69	70-74	75-79	80-84	Total 10-84	Ratio of brides to total
15-19	9	3,302	1,395	124	17	3	2									4,852	1,608
20-24	44	23,130	56,029	11,302	1,437	325	60	22	4	1						92,354	30,603
25-29	18	10,637	50,597	34,896	6,739	1,369	282	78	20	1	1	1				104,639	34,673
30-34	1	2,795	15,513	17,366	9,130	2,476	525	146	26	4	1					47,983	15,900
35-39	3	917	5,134	7,298	5,672	3,621	1,038	*313*	65	15	2	2				24,080	7,979
40-44	1	237	1576	2,564	2,811	2,473	1,502	510	112	26	8	1				11,821	3,917
45-49	2	115	598	1,077	1,313	1,653	1,279	859	263	74	36	8				7,277	2,411
50-54		41	183	384	538	768	754	675	406	117	37	20	2	1		3,926	1,301
55-59		11	73	129	197	313	360	445	289	218	65	26	4	2		2,132	706
60-64		6	28	71	79	152	162	207	208	144	106	60	16	2	1	1,242	412
65-69		1	15	24	43	66	80	133	122	113	105	97	19	7	1	826	274
70-74			6	16	17	30	50	47	65	41	50	59	28	6		415	138
75-79		1	2	3	8	6	11	13	17	31	14	21	25	8	4	164	54
80-84			2	2	2	2	8	10	7	4	9	4	8	4		62	21
85-90				1			1	4	1	1	1	1	1		1	12	4
Total*	78	41,193	131,151	75,257	28,003	13,257	6,114	3,462	1,605	790	435	300	103	30	7	301,785	100,000
Ratio of Brides to total	26	13,650	43,458	24,937	9,279	4,393	2,026	1,147	532	262	144	99	34	10	2	100,000	0.3313617

* Brides over 85 and bridegrooms over 95, and unspecified cases are omitted.

The bordered numbers denote the maximum on the vertical lines;

The shadowed numbers denote the maximum on the horizontal lines

! The values corrected for mistatement of ages 18, 19, 20 and 21 give the following results: for 3,302 and 1,395, 3502 and 1,481; and for 23,130 and 56,029, 23,172 and 55,701. In the totals 41,193 and 131,151 become 41,435 and 130,909; and 4,852 and 92,354 become 5,138 and 92,068. The ratios 13,650 and 43,459 become 13,730 and 43,378; and 1,608 and 30,602 become 1,703 and 30,508.

'0.3313617' - Factor of reduction to 100,000

Source: Knibbs, 1917: 199

Were these data smoothed, ‘they would give the probabilities of a marriage occurring within the year groups of specified ages or specified quinquennia’ (Knibbs, 1917: 198). Table 1.7.1 is discussed more fully below.

Knibbs moved on to a more sophisticated mathematical discussion on the ‘Frequency of marriage according to age representable by a system of curved lines’:

> Frequency according to pairs of ages (bride and bridegroom) can best be represented by a surface, the vertical height of which, above a reference plane, is the frequency for any pair of ages denoted by *x, y* co-ordinates. The numbers marrying in any given period, whose ages range between $x - \frac{1}{2}k$ and $x + \frac{1}{2}k$ (for bridegrooms), and between $y - \frac{1}{2}k$ and $y + \frac{1}{2}k$ (for brides), as ordinarily furnished by the data, are denoted by *Z*, the height of the parallelepiped. This frequency may, of course, be expressed as for the *exact age*, or it may be for the *age-groups.* When *k* is not infinitesimally small, the difference between the two is *sensible* and *important*. We shall assume for the present that the frequency varies only with age (*x*) in question, instead of being of various ages between $x - \frac{1}{2}k$ and $x + \frac{1}{2}k$. The age-group frequency denotes the frequency with the ages distributed between the limits referred to (Knibbs, 1917: 199).

For most practical purposes, Knibbs continued, the age-group frequency is the most important. Hence, supposing the exact frequency, *z*, for the population *P*, to be $\frac{z}{P} = F(x.y)$ Knibbs proposed the following equation for any group-value:

$$Z = P \iint F(x, y)dxdy\,.$$

To my knowledge this formula was the first two-sex mathematical equation ever proposed in demography in any treatment on the interaction between the sexes. Important aspects of the demographic debate upon marriage, since the late 1940s and from a two-sex point of view, are explicitly touched on in *The Mathematical Theory of Population.* For instance, Schoen’s (1988) recent review of the mathematical theory on the interaction between the sexes discussed the concept of 'magnitude of marriage attraction' and the properties of its harmonic mean solution:

> Let us focus on marriage and articulate the analogous two-sex population concept, the *magnitude of marriage attraction,* which reflects the mutual attraction for marriage between males and females independently of the age-sex composition of the population. The magnitude of marriage attraction differs from the force of decrement to marriage because the force only relates to the behaviour of both sexes (Schoen, 1988: 121).

In his theory on nuptiality, Knibbs discussed the frequency of marriages according to age representable by a system of curved lines; he referred to the errors in dealing with group-ranges and in contrast to the central value of the range of ages proposed to compute the 'weighted mean' of the differences of the groups adjoining on either side of an age group.

However, the depth of Knibbs's two-sex approach on nuptiality can be better grasped when placed in the context of what he called the gamic conditions; a subject discussed in the last part of Chapter 12 of the *Mathematical Theory of Population.* It was here that Knibbs set his theory of probability of marriages in age-groups and applied to what he called the 'protogamic surface' (Knibbs, 1917: 214-228) and the 'gamic surface' (Knibbs, 1917: 228-231).

The gamic conditions: 'General theory of protogamic and gamic surfaces'

Following the definition of the equation *Z* Knibbs moved on to focus on the subject from an empirical point of view and searched for appropriate ways to represent the interaction between bridegrooms and brides statistically:

> The ages of husbands being adopted as abscissae, and those of wives as ordinates, the infinitesimal number *dM* in an infinitesimal group of married couples, consisting of husbands, whose ages lie between *x* and *x+dx*, and their wives, whose ages lie between *y* and *y+dy*, will be:
>
> $$dM = Zdxdy = kF(x,y)dxdy$$
>
> Thus $Z = kF(x,y)$ is represented by a co-ordinate vertical to the *xy* plane. Since *Z* denotes an actual number of persons in a double age-group, between say the earliest age of marriage and the end of life, viz., $(x_1$ to $x_2)$ and $(y_1$ to $y_2)$, it is necessary, if we desire to institute comparisons between different populations, that *Z* should be expressed as a *rate*, *z* say: that is, *z*= either *Z/P;* or *Z/M*; that is to say, the vertical height will represent the relative frequency of married couples whose ages are, in the order of husband and wife, *x* and *y*, in either the whole population *P*, or the married portion of it *M*. Thus we shall have
>
> (418) .... *P*, or $M = k\iint F(x,y)dxdy$.
>
> If the value of the double integral be taken for the limits denoting the range of ages of the married, say about 11 to 105, we shall have either *M/P*, or unity, as the result; according as we denote by frequency in reference to the total population or to the total married (Knibbs, 1917: 201-202).

In this way Knibbs set, for the first time, the concept of 'conjugal potential', which today is best known as 'marriage function' or, more generally, 'mating function'. The 'marital or gamic condition of a community', Knibbs explained,

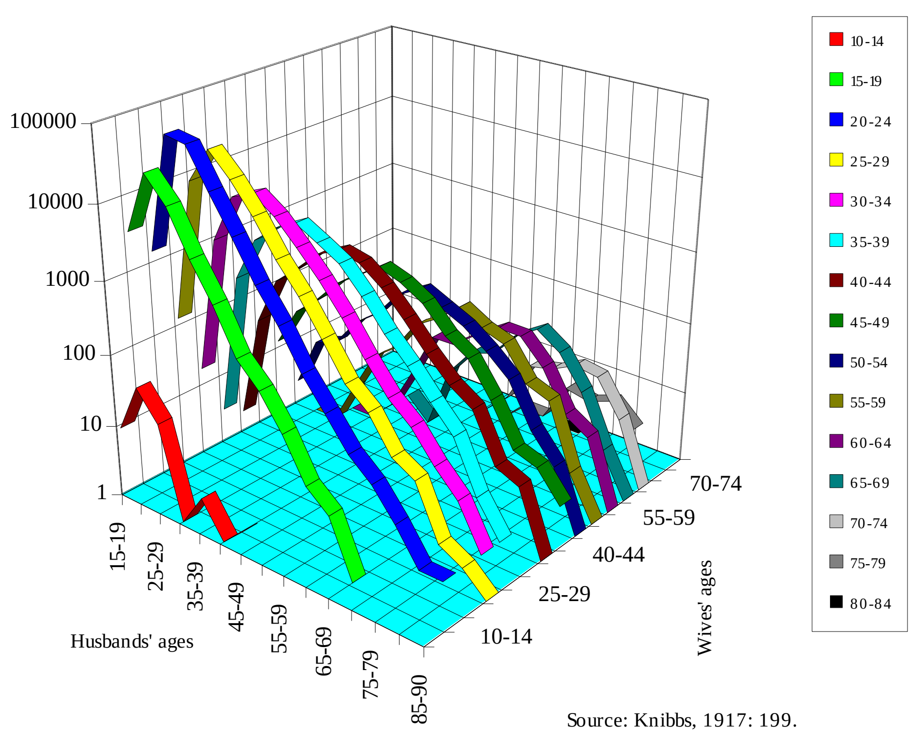
> is completely specified by the gamic surface *F(x,y,z)*, where the frequency for any pair of ages of bridegrooms and brides is denoted by *x, y* co-ordinates (the 'gamic meridians), and *z* corresponds to the exact frequency of marrying numbers denoted by *Z* ... The values of *x, y, z* for the unique mode of the surface may be called the *gamic mode of the 'population'*, or of *'married population'*, according as the constant *k*, in (418) above, gives *M/P*, or unity for the value of the double integral between the widest age limits (Knibbs, 1917: 202).

In addition, the 'gamic characteristics' of the population are more briefly, though less completely, defined by two factors: (1) the 'gamic meridians', that is the two principal meridians defined by the line joining the modes of the curves *x*=*a* constant and *y*=*a* constant and passing through the unique mode, as well as the curve *z*=any constant passing through the unique mode; (2) the position (and magnitude) of the *gamic mode*.

The term 'gamic' may only be used currently in social sciences as part of words such as 'monogamic', the habit or practice of having only one mate, or poligamic in case more than one mate is involved. Etymologically 'gamic' refers to sexual (opposed to 'agamic') and comes from the Greek *gamikós*, 'of or for marriage' (*Macquarie Dictionary*, 1985: 724). Knibbs identified two types of surfaces: the protogamic surface, referring to the frequency of marriage at particular pairs of ages, and the gamic surface, referring to the frequency of the number of persons of particular pairs of ages living together in the state of marriage.[5] Later in the text Knibbs (1917: 224) explained with regard to conjugal age-relationships that the protogamic age-relationships may be ascertained from marriage records. In turn, the gamic age-relationships refer to the instantaneous relationships at any moment and are disclosed by a census. Most of the analysis on gamic conditions is focused on the protogamic surface.

Since the characteristics of the protogamic surface are disclosed by the position of the maximum points, Knibbs returned to the data which are depicted in this thesis by Table 1.7.1 and are graphically represented in Figure 1.7.2.

Figure 1.7.2 Number of marriages arranged according to age at marriage, Australia, 1907-1914.



Source: Knibbs, 1917: 199.

[5] In a footnote, Knibbs explained:
The word 'isogamy' has already been appropriated in a different sense in biology, viz., to denote the union of two equal and similar 'gametes' in reproduction. This, however, will obviously lead to no confusion. The isogamy of a people might be regarded as of two kinds, *initial* or *nuptial* isogamy (isoprotogamy), and *characteristic* or *marital* isogamy (or simply isogamy) (Knibbs, 1917: 202).

As Table 1.7.1, shows the numbers of marriages corresponding to any given age for brides (the columns) show a clearly-defined maximum value; but the corresponding numbers of marriages to any given ages for bridegrooms (the rows) in many cases show two or even three maximum values. In this latter case, too, the maximum is often less clearly defined. Knibbs indicated two ways of estimating the position and frequency at the maximum (or any other point):

> One is to ascertain the position and frequency for the maximum of the frequency integral taken over the range $x-\frac{1}{2}$ to $x+\frac{1}{2}$, or over the range $y-\frac{1}{2}$ to $y+\frac{1}{2}$; the other is to determine those elements for the maximum instantaneous frequency ; that is to ascertain the point when the frequency for an indefinitely small range is a maximum (expressed, however, per unit of age-difference, say one year) (Knibbs, 1917: 204).

By applying some formulas introduced in Chapter 7 of the *Mathematical Theory of Population,* Knibbs (1917: 204-211) calculated the position and value of the maximum points, those on the surface for ages of brides constant, those of bridegrooms being variables, or for ages of husbands constant and those of brides variable.[6] The highest point surface derived for the group bridegrooms was about 23.4, and for brides 21.6 years of age; the frequency attaining to about 4,200, or about one seventy-second part (0.013911) of all marriages (Knibbs, 1917: 207).

At this stage Knibbs was hardly satisfied with his results, and admitted their uncertain because of the abnormalities related with misstatement of the age at marriage. In another clear demonstration of his grasp of the difficulties faced when one attempts to deal with aspects of the complementarity between the sexes he remarked:

> It is, of course, much to be regretted that social organisation does not admit of the social-psychological fact of conjugal frequency at equal and disparate ages being accurately ascertained (Knibbs, 1917: 208).

Knibbs did not give up to the subject here, after expressing his regret for adversities of social organization. On the contrary, rather than using this as an excuse to abandon the matter, Knibbs moved on immediately to search for feasible directions aiming to overcome the difficulties he faced. First, Knibbs (1917: 211) admitted that 'For sociologic purposes, a table shewing the relative marriage frequency in various age-groups is of obvious importance'. So, Knibbs took the married and unmarried Australian population by age-groups, from 1907 to 1914, he deduced the relative frequency of marriage for an estimated 1,000,000 marriages as in Table 1.7.2.

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[6] For details of the results that Knibbs computed see Knibbs, 1917: 204-205.

Table 1.7.2 Relative frequency of marriage in various age-groups, Australia, 1907-1914

Bridegroom's Age	Brides' age 10-14	15-19	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65-69	70-74	75-79	80-84	85-89	All ages 10-89
15-19	30	11,605	4,920	411	56	12	7	3	2	1							17,048
20-24	146	76,788	184,576	37,452	4,762	1,077	199	73	13	3	1						305,080
25-29	60	35,249	167,668	115,639	22,331	4,537	935	259	66	6	3	2					346,765
30-34	10	9,262	51,407	57,547	30,255	8,205	1,740	484	86	13	7	3					159,019
35-39	7	3039	17,013	24,184	18,795	11,999	3,440	1,037	215	50	13	5					79,797
40-44	5	785	5,222	8,496	9,315	8,195	4,978	1,690	371	86	30	10	1				39,183
45-49	4	381	1,982	3,569	4,351	5,477	4,239	2,827	872	245	80	27	3				24,057
50-54	3	136	607	1,273	1,783	2,499	2,545	2,237	1346	388	166	53	7	3			13,046
55-59	2	43	182	414	686	978	1,293	1,425	1027	697	215	99	17	6	2		7,086
60-64	1	20	93	209	331	457	547	686	689	524	351	199	50	9	3		4,169
65-69	1	7	43	88	143	219	265	365	431	431	315	182	63	13	5		2,571
70-74	1	5	23	40	66	99	146	186	215	215	166	113	73	21	7	1	1,377
75-79	1	3	7	13	20	28	38	48	64	85	92	73	47	27	11	1	558
80-84		1	6	9	10	14	22	28	33	29	23	13	8	4	2	1	202
85-90			1	1	2	3	5	10	8	5	3	2	1	1			42
Total	271	137,324	433,750	249,345	92,906	43,799	20,398	11,358	5,438	2,778	1,465	781	270	84	30	3	1,000,000

Source: Knibbs (1917). *The Mathematical Theory of Population, p. 211*

The totals in the final column of Table 1.7.2, entitled 'All ages 10-89' are about ten times those in the final column of Table 1.7.1. Though in substantial agreement, the totals in the two tables are not absolutely identical because the results in Table 1.7.1 have been slightly smoothed. In the end, based upon the marriages of the 8-year period, 1907 to 1914 inclusive, Knibbs found a middle point of time to be 0 January 1911, while the census was 3 April 1911. The total marriages were 301,922 or about 37,740 annually; half of them had occurred by about April 28, 1911, that is 25 days after the census.

A second solution for the difficulties in dealing with both sexes was more of a theoretical and mathematical nature. Knibbs took the smoothed results of the census just described, the computation of the unmarried at each age, the estimate of ratios of the males to the females (M/F), and the masculinities of the various age-groups, which were required 'hereafter for the computation of the probability of marriage according to pairs of ages' (Knibbs, 1917: 212).

Knibbs's 'theory of the probability of marriages in age-groups'

Regardless of the lack of data for a definite and rigorous determination of the probability of marriage in age-groups, Knibbs commented: 'a fairly accurate estimate is possible by means of a somewhat empirical theory' (Knibbs, 1917: 214).

> Suppose that in any age-group there are M unmarried males and F unmarried females; and that in a unity of time N pairs of these marry. The probability *with F females in the group*, of a particular marriage occurring among the M males is obviously N/M; and *with M males in the group*, the probability of a particular marriage occurring among the F females is similarly N/F. Such a statement of probability, however, lacks generality. To obtain a more general one, an expression is needed which, given a definitive tendency towards the conjugal state in males and in females, though not necessarily of the same strength (or

> potential) in each sex, and not necessarily independent of the relative numbers of the sexes, nor even independent of the lapse of time, will give the number of marriages occurring in a group, constituted in any manner whatever in regard to the numbers of either sex. We shall call the tendency to marry the *conjugal potential* under a given condition. In the case of males let the conjugal potential be denoted by γ, and in the case of females by γ'; γ and γ' vary with age, doubtless also with time, and (we may assume) with the relative frequency of *M* and *F*. (Knibbs, 1917: 214).

Then Knibbs discussed the specific conditions of application of the 'conjugal potential': (1) when the conjugal potential is assumed to vary somewhat as some constant; (2) when the numbers of unmarried of either sex are equal or not; (3) if the conjugal potential vary with age; and (4) assuming that the marriage of particular pairs is equally probable, and that the relative magnitude of M and F does not influence the probability, *p*. In addition, he proposed some additional conditions, those which should lead to the expression that will readily 'enable the number of marriages likely to occur in each age-group to be computed when the numbers of unmarried males and females in the group are known' (Knibbs, 1917: 214). Thus, *q* considered the tabular number, the number of marriages, *N*, could be computed by means of the following formula:

$$N_{xy} = q_{xy} \cdot M^{\frac{F}{M+F}} \cdot F^{\frac{M}{M+F}} = q_{xy} \cdot M\phi^{\frac{1}{1+\phi}} = q_{xy} \cdot F\mu^{\frac{1}{1+\mu}}$$

From here, to find *q* from the results furnished in his tables of unmarried males and females and the masculinity at each year of age as well as for computing the effect of unequal numbers of unmarried males and females on the frequency of marriage Knibbs proposed:

$$\log q_{xy} = \log N_{xy} - \frac{1}{1+\mu}\log M - \frac{1}{1+\phi}\log F$$

x and *y* denoting the central values of the age-groups, that is $x \pm \frac{1}{2}k, y \pm \frac{1}{2}k$ where *k* is the range of the group. In order to be more easily applicable Knibbs proposed the following simplification:

> Let $S_{xy} = M_x + F_y$, that is, let S_{xy} denote the total number of single persons in the groups of males of age *x* and females of age *y*, and let the masculinity (or femininity) of S be denoted by M/F (or F/M); then assuming that the probability is identical for *A* males and *B* females, with that for *B* males and *A* females (which, however, though by no means certain, is not determinable from existing data) we may compute the value of the ratio
>
> (433).... $R_\mu = R_\phi = (M^{\frac{F}{M+F}} \cdot F^{\frac{M}{M+F}}) / \frac{1}{2}(M+F) = F\mu^{\frac{1}{1+\mu}} / \frac{1}{2}S = M\phi^{\frac{1}{1+\phi}} / \frac{1}{2}S$
>
> which depends merely upon the masculinity, μ (or the femininity ϕ), and is independent of the absolute value of *S*, or of *M* and *F*. Consequently with a table of values of *R* arranged according to the argument μ (or ϕ),we have, by simply dividing *M* by *F* , (or *F* by *M*) and entering the table,
>
> (434) .... $N_{xy} = \frac{1}{2}S_{xy} \cdot R_\mu \cdot q_{xy} = say \frac{1}{2}S_{xy} \cdot Q_{xy}$
>
> *Q* itself could be tabulated but for the fact that the masculinity in age-groups may differ appreciably with the lapse of time.
>
> (Knibbs, 1917: 216-217)

Knibbs then calculated two tables, one for *R,* depending upon the masculinity (or femininity), and the other for *q* depending on the frequency of marriage for the age-groups in question. 'After preparing the table of values of *R,* those of *q* can readily be calculated', Knibbs (1917: 217) remarked. Moreover, he concluded that in using the values of *R*, it is 'a matter of indifference whether it the argument 'masculinity' or 'femininity' when determining the frequency of marriage for the age-groups in question.

Following the computation of the masculinity of the unmarried for any combined age-groups Knibbs established the 'probability of marriage according to pairs of ages' as follows:

> Assuming that the 'conjugal potential' does not change in any community, the number of marriages likely to occur among groups of the unmarried of given ages can be computed by means of formula (434) ... If the conjugal potential are the same for *A* males and *B* females as for *B* males and *A* females, and the law of variation is, as by hypothesis,
>
> $$(437) \ldots\ldots (\gamma + \gamma^{'})\alpha M^{\phi_2} \cdot D^{\mu_2} = M\phi_1^{\mu_2} = F\mu_1^{\phi_2}$$
>
> then the qualification as to masculinity being approximately identical disappears. It is not unimportant, however, to remember, that the fundamental assumption would have to be very erroneous (and that would seem to be impossible) in order to seriously prejudice the precision of the result obtained by the application of the formula (434). The error in any real application of the formula can be a differential one only, and if the constitution as regards numbers of the population be approximately therefore that from which it was derived, any defect in the theory of variation with relative numbers of the sexes, formula (430), has no sensible effect (Knibbs, 1917: 223).[7]

Even by current standards, Knibbs's reasoning remains highly sophisticated and complex; among other things, the formal and empirical aspects of theoretical issues were faced with equal seriousness.

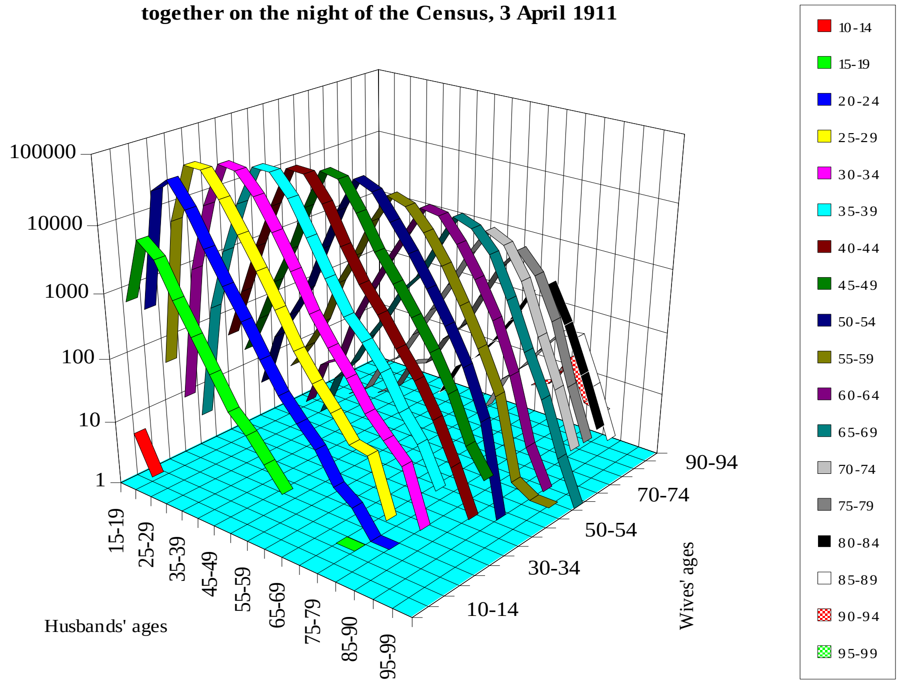
The non-homogeneous groupings

By taking the total number of 616,738 married persons living together whose ages were fully specified, and who were living together on the night of 3 April 1991, Knibbs computed a table of numbers of married persons per 1,000,000 married couples in five-year age groups. The results are shown graphically in Figure 1.7.3.

---

[7] γ denotes the conjugal potential in the case of males and $\gamma^{'}$ in the case of females. μ_1 and ϕ_1 are the same as in formula (433) and refer, respectively to masculinity and femininity; μ_2 and ϕ_2 correspond also to the masculinity and femininity but are drawn from another method which Knibbs (1917: 132) discussed earlier in the chapter on 'masculinity of population'.

Figure 1.7.3 Number of married persons per 1,000,000 married couples, living together on the night of the Census, 3 April 1911



Source: Knibbs, 1917: 224.

These data prompted Knibbs to point out that in calculating the age-groups the sex taken as argument is not irrelevant. The results differ if the age of the husband instead of the wife is used and they 'have no obvious direct mutual relation' (Knibbs, 1917: 224). In this and other 'analogous groupings of a non-homogeneous character', Knibbs admitted that a one-sex approach may be more adequate:

> In cases of the kind under consideration two formulae are needed; in one the argument is the age of the husband (or bridegroom), in the other the age of the wife (or bride) (Knibbs, 1917: 224)

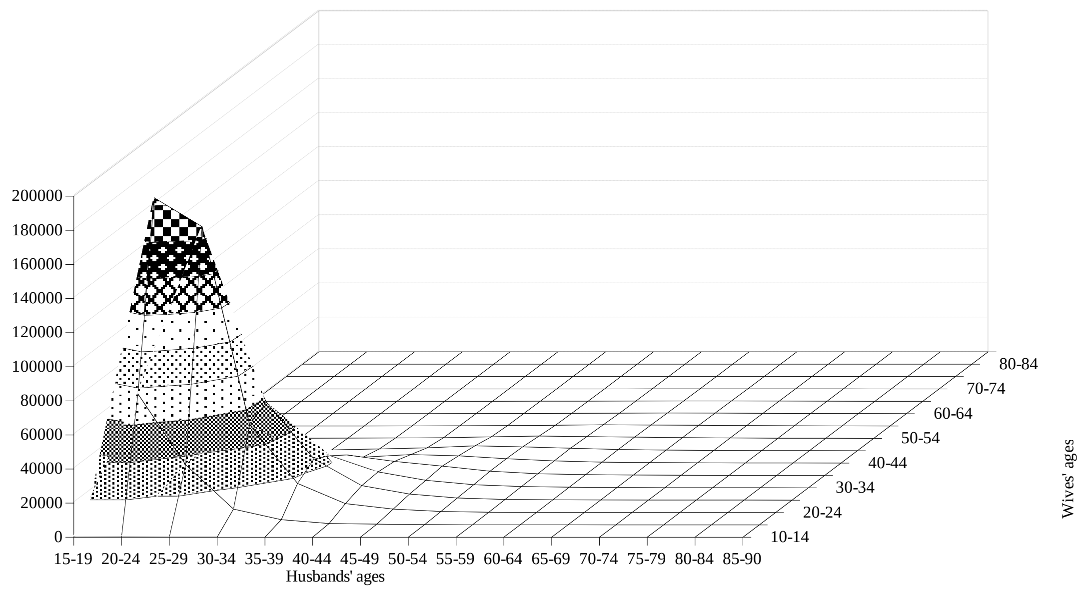
Upon the non-homogeneous groupings of data Knibbs remarked about the differences in the results based on the argument *x* (husband) as compared those based on the argument *y* (wife):

> If the distribution about the mode in such cases be not symmetrical in each, in fact *if it be not similar in all respects*, no direct functional relationship subsists between results for groupings arranged according to the values of *x*, and those for groupings arranged according to the values of *y*. Groupings subject to this limitation may be called *non-homogeneous groupings*, and require special consideration (Knibbs, 1917: 225).

Knibbs specified the average differences between ages of husbands of any age and the average ages of their wives, and vice-versa between the ages of wives and the average ages of their husbands.

Based on the same data used to draw Figure 1.7.4 Knibbs constructed the gamic surface, on the same principles applied to the construction of the protogamic surface. Figure 1.7.4 depicts the protogamic surface chart, which can be compared with the gamic surface chart in Figure 1.7.5. These images are intended to show the reader the level of complexity of Knibbs's formal analysis, which included not just elaborate mathematical reasoning but also geometrical and graphical reasoning.

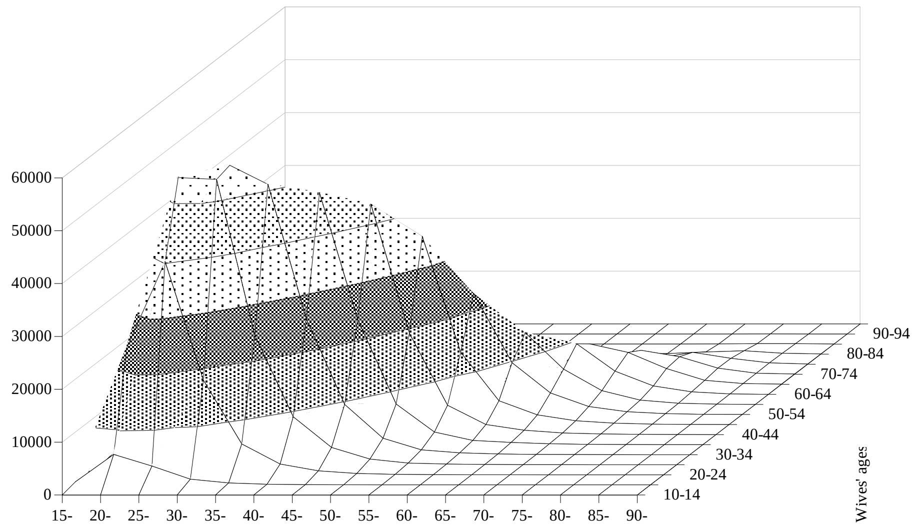
Figure 1.7.4 The protogamic surface, Australia 1907-1914



Source: Knibbs, 1917: 211

In Figure 1.7.4 the isoprotogams are less elliptical and regular than the isogams in Figure 1.7.5. The interpretation of the curves for isogams is, *mutatis mutandis*, the same as that for the isoprotogams. However, while in Figure 1.7.4 the data apply to persons 'living in the state of marriage', in Figure 1.7.5 the data apply to 'persons at the moment of marrying' (Knibbs, 1917: 228).

Figure 1.7.5 The gamic surface, Australia 1907-1914



Source: Knibbs, 1917: 224

Knibbs finished his Chapter 12 with a summary on the two gamic conditions in his theory of nuptiality: (1) the protogamic norm or nuptiality, based on the aggregation of the marriages of a large number of people; (2) the conjugality or gamic condition based on the census results. He added that the protogamic norm should reflect the trend in regard to the early institution of marriage, while the gamic norm should reflect the modification of this by factors such as the change in longevity and the frequency of divorce. These norms could include the curves of the totals according to the age of the males (bridegrooms and husbands), and according to the age of the females (brides and wives), as well as the frequency of the group-pairs. Likewise, the norms of conjugal states such as 'never married', 'divorced', and 'widowed', might also give the frequencies according to group-pairs. Knibbs concluded Chapter 12 writing:

> the probability of marriage depends, among other things, upon the relative numbers among the unmarried of the sexes. So long, however, as a population does not greatly change its constitution according to sex and age, the crude probability of marriage according to sex and age may be regarded as varying approximately as the annual rate. This probability may be called *pheithogamic*[8] *coefficient* for the sex and age in question (Knibbs, 1917: 232).

The monogenous approach: fecundity, sterility and fertility

Following the previous discussion on natality and nuptiality, Chapters 13 and 14 of *The Mathematical Theory of Population* deal with reproductive efficiency through the

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[8] From the Greek term meaning ‘to prevail upon’, the Goddess of Persuasion, and of or for marriage (Knibbs, 1917: 232).

concepts of fecundity and fertility. The phenomena which directly concern the measure of the reproductive power, one can read at the beginning of Chapter 13,

> are in general complex, the variation of the reproductive power being in part of physiological origin, and in part of the result of the reaction of social traditions upon human conduct. This will appear in any attempt to determine the laws of what has been called bigenous (better, digenous) natality, or natality as affected by the ages of both parents, as distinguished from those affecting merely monogenous natality, or natality as related to the producing sex (Knibbs, 1917: 233).

The review of Körösi's paper provided above should allow the reader to trace the origin of the framework of reproduction sketched in this quotation around the two operational concepts digenous (Körösi called it bigenous) and monogenous fertility. Like Körösi and perhaps all demographers at the turn of the nineteenth to the twentieth century, Knibbs was mostly concerned in establishing the measurement, the methodology and description of fertility as a demographic output. Perhaps the main difference of these two demographers, as compared with others such as Böckh, Lotka and Kuczynski, is that the latter did not hesitate in reducing the study of reproductive efficiency to the producing sex only. As Kuczynski (1935: 206) put it, ‘Since we are concerned here with births only, it suffices to take into account the female population’.

Throughout Chapter 13 and, above all, Chapter 14, Knibbs returned, time and again, to the interaction between the ages of both sexes and the role of nuptiality. Contrary to Körösi, Knibbs did not restricted his analysis of the census data to an empirical one; he also provided a brief but systematic insight on the formalization of demographic reproduction. In deducing the most probable value for certain demographic phenomena, Knibbs remarked, it will be necessary first to minimize the effect of misstatement of age; and secondly, to treat demographic reproductive efficiency as a derivative and dependent ‘upon the age-distribution and conjugal condition of the producing sex’ (Knibbs, 1917: 233). The following sentence indicates Knibbs’s view as to the place of an analysis of fertility from a one-sex and a two-sex perspective:

> Many questions concerning the measurement of fertility and fecundity can be settled with sufficient precision without recourse to a differentiation depending on the age of the father, the better in Australia, perhaps, inasmuch as the decay of virility with the age is not well marked, and in this aspect the digenous fertility stands in marked contrast with that of Hungary ... *Digenous fertility and digenous fecundity* will denote the fertility and fecundity of the female, as modified by the age of the associated male, and therefore is considered in relation to the ages of both males and females. Consequently computations of monogenous fertility or fecundity will be based upon the age of the female (Knibbs, 1917: 233).

It is important to recall an aspect already mentioned in Chapter 6 regarding Knibbs’s definition of the terms 'fertility’ and ‘fecundity’; he used these terms in the reverse way from the current English usage, that is in the same way they are currently applied in Latin languages, such as French, Italian, Spanish and Portuguese. To avoid confusion with the remainder of this thesis, the references made here to fertility and fecundity correspond to their usage in English in contemporary times and this is opposite to the way found in Knibbs’s *Mathematical Theory of Population*.

Table 1.7.3 includes Knibbs's compilation of the available methods of measuring reproductive efficiency, which he saw as being ' all more or less defective'; he concluded: 'A more satisfactory scheme is to construct a monogenous age-group "natality table" for married, and one for unmarried, females' (Knibbs, 1917: 236). Even in this case, Knibbs was not completely satisfied: 'It is, however, not perfectly satisfactory, because, as already indicated, it would appear that the age of the father as well as that of mother affects the probability of maternity' (Knibbs, 1917: 136).

Before returning to the effect of father's age upon the probability of maternity, Knibbs went first through a lengthy and detailed analysis of a variety of issues focused on the monogenous female only.[9] In particular, about the 'theory of fecundity, sterility and fertility', Knibbs remarked:

> The *fertility-ratio* [read fecundity-ratio] or *probability of maternity in a unit of time* may be defined as the proportion of cases, which, subjected to a given degree of risk for a unit of time, result in maternity; and similarly, the *sterility ratio* or *probability of maternity* is the arithmetical complement of the probability; or calling these respectively p and q, $p+q = 1$ (Knibbs, 1917: 319).

In 1977, Smith and Keyfitz claimed that Corrado Gini (1924) was the first to explore the distinctions and implications of the fact that pregnancy and birth distributions are mathematically separated by interval of non-risk. If this statement is true it should be only as to Gini's proposal 'that birth intervals be treated as waiting time problems dependent on fecundability' (Smith and Keyfitz, 1917: 365). However, by reading *The Mathematical Theory of Population* it becomes apparent that Knibbs already raised and debated most of the 'probability models of conception and birth' later developed and formalized by authors such as Louis Henry (1953), Basu (1955), Tietze (1962), Potter (1963), and Sheps (1964) (see Smith and Keyfitz, 1977: 365-395). Among the issues addressed by Knibbs's work are the following: probability of a first birth occurring within a series of years after marriage (p. 245); maximum probability of a first birth (p. 248); positions of average intervals for groups of all first-births (p. 267); range of gestation period (p. 276); and proportion of births attributable to pre-nuptial insemination (p. 278).[10]

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[9] Some of such issues were the following: norms of population for estimating reproductive efficiency and the genetic index; the natality index; age of beginning and of ending; the maternity frequency, nuptial and ex-nuptial, according to age, and the female and male nuptial-ratios; maximum probabilities of marriage and maternity; maximum probabilities of first-birth; the nuptial and ex-nuptial protogenesis; initial and terminal non-linear character of the average issue according to duration of marriage; crude fecundity; secular trend of reproductivity; crude and corrected reproductivity; theory of fecundity, sterility, and fertility; fertility according to age and duration of marriage (Knibbs, 1917: 136-344).

[10] The investigation on fecundability lacks much investigation even today. Gray (1995), in a recent seminar at the ANU, presented his work-in-progress entitled 'Returning to fecundability'. Gray attempted to measure the strength of fecundity through a measure of fecundability or the probability of conception in the length of prospective birth intervals after the end of whatever periods of amenorrhoea and abstinence from sexual relations were reported by the respondents in the survey; he applied this to data from the Demographic and Health Survey carried out in Indonesia in 1991.

Table 1.7.3 Knibbs's compilation of methods of measuring reproductive efficiency			
Rate measured by		**Deduced result know as**	**Remarks**
Numerator	**Denominator**		
Total births, *B*	Total population, *P*	Crude birth rate, B/P	Is dependent on age, sex, and conjugal constitution of total population, and therefore not strictly comparable as between different populations; it measures merely one element determining increase.
Total births, *B*	Total female population, *F*	Birth-rate referred to total number of women, B/F	Is dependent on female population only and is affected of course by the age and conjugal conditions of that population.
Total births, *B*	Female population of reproductive age (viz., from about 10 to 60), *F* , say	Birth-rate referred to women of reproductive age only , B/ F	Indicates reproductive efficiency of all women within the reproductive period. Owing, however, to the limits of this period being ill-defined at the initial and terminal ages, to the largeness of the number of women at those ages, and to the fact that it is independent on the age-constitution within the group chosen to represent the reproductive age, the rate is not as definite as is desirable. The denominator, however, is a good crude measure of the potential of reproductive efficiency of the population.
Births in each age-group, B_X	The women in same groups, F_X	Birth-rate referred to women of each age-group in question, B_X/ F_X	Is uncertain for comparison because the ratio of married to unmarried women may vary, and the relative frequency of maternity in each is not identical.
Nuptial births in each age-group of unmarried women, B_x^m	Married women in same groups, M_X	Nuptial maternity rate for each age-group, $\frac{B'_x}{M_x}$	Shows only the average frequency of maternity (average probability of maternity) for married women in each age-group.
Ex-nuptial births in each age-group of unmarried women, B_x^m	Unmarried women in age-group, U_X	Ex-nuptial maternity rate for each age-group, $\frac{B_x^{m'}}{U_x}$	Shows only the average frequency of maternity (average probability of maternity for married women in each age-group.
Appropriately weighted sum of birth-rates of the married and unmarried	Unity	Modified 'Nuptial Index of Natality'	This attributes the reproductive facts of an existing population to a supposititious 'standard' population, in which the relative number of married and unmarried females is the general average (norm) for the groups of populations to be compared. The comparison so attained may be regarded a suitable comparative measure of reproductive efficiency (natality).
Source: Knibbs, 1917: 236			

Complete versus partial tables of fertility: the digenesic approach on fertility

Still in reference to the theory of fecundity, sterility and fertility, Knibbs asserted that the 'degree of risk' of fecundity not just decreases after a certain age of women, but it also 'varies with the age of the husband' (Knibbs, 1917: 319). Yet, in some countries fecundity may vary but slightly with the age of husband, Knibbs acknowledged. Hence, by ignoring the issue of age of husband, Knibbs proposed that in place of complete tables of fecundity and fertility, partial tables may serve 'all general practical purposes' (Knibbs, 1917: 320). Table 1.7.4

summarizes and compares the information required by complete and partial tables of fecundity and fertility.

Table 1.7.4 Complete versus partial tables of fecundity, sterility and fertility	
Arguments of complete tables	Arguments of partial tables (i.e. ignoring the effect of husband's age)
(i) Age of wife, with (ii) age of husband (iii) Duration of marriage	(i) Age of wife only (i.e. with husbands of all ages) (ii) Duration of marriage
	Knibbs, 1917: 320

In case of fecundity and sterility, the tables should show, Knibbs proposed, for each combination of age and duration of marriage, the proportion of married women who have born one child. Likewise, in case of fertility, the tables should show, for each combination of age and duration of marriage, the proportion of married women who have born *n* children, where *n* referred to the successively parity 0, 1, 2, 3, 4, etc.

With regard to the 'digenesic surfaces and diisogenic contours', Knibbs explained:

> If the husband's age be not ignored fecundity (read fertility) relations become greatly increased in complexity. For example, instead of a maternity rate or a birth-rate according to the age of wife, we have a series for each age of the husbands; the compilation-table becomes one of double entry, and the various fertility and fecundity-relations become correspondingly multiplied (Knibbs, 1917: 349-350).

The reasoning displayed by this statement corresponds to that of Körösi. Knibbs reviewed the issues treated by Körösi, but added to the subject his much more sophisticated and formal reasoning. From his theoretical discussion and then the comparison of the results for Australian population and those provided by Körösi for the population of Budapest, Knibbs made two important inferences. First, that for a given difference of age in the wife, the equivalent difference of age in the husband is not the same. To make one equal the other, Knibbs proposed to introduce a factor called 'the masculine factor of age-equivalence'. And vice-versa, to make the difference in the wives' age equal, for a given difference in the age of husband, a factor called 'the feminine factor of equivalence' should also be need.

The second inference was more a generalization on diisogeny drawn from the comparison of the results of the diisogeny in Australia with the diisogeny in Budapest:

> For ages greater than that of the maximum fertility of women and for those combinations of ages of husband and wife which are most common, the fertility-ratio may be regarded as represented - very roughly of course - by straight lines: that is to say, x and y being respectively the ages of husband and wife at the time of the birth, the fertility-ratio is constant when $kx+y$ is constant ...The pairs of ages, x and y, which give identical fertility-ratios, may be called *corresponding age-pairs* ... Moreover the *fertility-ratio* (and thus the value of k) *diminishes with increase of the sum of the corresponding age pairs* (the age of maximum value having been passed. Obviously, also, k differs for various populations (Knibbs, 1917: 362).

Chapter 14 of *The Mathematical Theory of Population* finishes with a discussion on six issues which Knibbs followed in his subsequent work during the 1920s: multiple ‘diisogeny’, that is the equal frequency of twins, or of triplets, etc., according to pairs of ages, the series of ages giving equal frequency being in this case also known as ‘corresponding pairs’; twin and triplet frequency according to ages; apparent increase of frequency of twins with age of husbands; triplet ‘disogeny’; frequency of twins according to age and according to order of confinement; unexplored elements of fertility. As to the latter issue and in conclusion of the analysis on fecundity and fertility, Knibbs remarked:

> To distinguish between the *effect of previous births* and age upon the frequency of maternity, of twins, etc., more comprehensive data are required than at present exist for Australia. The effect is one which, so far as the maternity-ratio is concerned, reflects social tradition in a larger measure than the physiological law; the latter is modified but not obliterated. In the case of twins, triplets, etc., the physiological laws doubtless alone operate (Knibbs, 1917: 369).[11]

Complementarity and two-sex demography: searching for a purpose

With this Chapter 7, I have concluded the review of the strands depicted in Figure 1.1. The six chapters included in Part I have placed the envisaged two-sex demography in the wider context of the development of demographic theory since its birth and earlier growth; they are expected to yield a valuable contribution to the development of a comprehensive two-sex perspective in three ways. First, the strands reviewed between Chapters 2 and 7 are consistent with the principle of complementarity between the sexes. In particular, they demonstrate that nothing could be more self-defeating for the development of a two-sex perspective than the idea that anything learnt elsewhere could be relevant to demography, even without finding any support in the history of its own ideas. Moreover, the chapters above reveal the utility of defining and following explicitly a guiding theoretical principle and avoid cutting adrift from the history of demography in general.

Secondly, the above historical review makes it clear that a coherent two-sex perspective can be developed in close association with the analytical bodies already in use in demography. However unstated and implicit are the theoretical principles in which conventional demography stands, none of its concepts, measures, methods and specific theories can be considered mindless and short of ideas. After all, even the most technical and formal tools in demography can and should be seen as part of specific analytical bodies used to study certain aspects of demographic reality.

There is a third and far-reaching valuable contribution that the historical review provided in Part I is expected to accomplish. I have tried to discuss in a logically coherent manner the strands depicted in Figure 1.1; this seems to be the best way to avoid bringing

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[11] *The Mathematical Theory of Population* contains four final chapters, 'Mortality', 'Migration', 'Miscellaneous', and 'Conclusion'.

together several concepts haphazardly and in an *ad hoc* fashion. The concepts in Figure 1.1 follow a sequence which is historically and theoretically consistent with the development of demography. Historically, the sex ratio seems to have been the first demographic measure ever created in the scientific study of population.

Chapters 2 to 4 have focused on the most simple measure of complementarity between the sexes and, in particular, revealed the Janus-like nature of the sex ratio: a measure of matters of fact and an explanatory resource in theory construction. Chapter 5 focused on the 'passion between the sexes', which Malthus used as an important demographic principle in the design of sexual reproduction and associated with reproductive mechanisms like 'marriage' and 'couple'. Chapter 6 is, perhaps, the pivotal chapter of this thesis because it has already raised the central issue in all this discussion: 'When, why and how should the complementarity between the sexes matter to demography?'. This question has never been adequately addressed by earlier demographers, not even in current times by demographers who have been interested in important research areas such as the 'determinants of fertility' and the 'two-sex problem'. Chapter 6 traces the evolution of the concept of fertility in demography revealing a periodization called 'three scientific breakthroughs in leaps of one hundred years'; this evolution is consistent with the three bifurcations depicted in Figure 1.6.1.

Chapter 7 reviewed expressions of the complementarity between the sexes such as nuptiality according to pairs of ages, the gamic conditions, the probability of marriages in age-groups, and the mating functions. These concepts have been reviewed in association with Knibbs's anticipation of a two-sex methodology. Knibbs perceived that demography had much to offer to the knowledge of population change even when the two sexes are studied separately from one another. But following Quételet and Körösi, Knibbs not only wondered about but discussed the feasibility of taking into consideration the role of both sexes in studies of population. Although Knibbs did not suggest any direction towards the conceptualization of fertility in the way proposed in Chapter 6, his *Mathematical Theory of Population* clearly anticipates the necessary condition for a two-sex demography: that for certain purposes the methodological frameworks should explicitly take into consideration the numbers and behaviour of both males and females.

This means that the strands illustrated in Figure 1.1 are mainly relevant to the definition of the necessary conditions for a two-sex perspective and finish where most of the two-sex methods usually start: nuptiality and mating functions. However, Chapter 6 already indicates that two-sex models should exist neither for their own sake, nor even to improve demographic measures that are reasonably produced on the basis of one-sex models.

Behind the idea on the three bifurcations in the development of the demographic concept of fertility lies the view that a two-sex approach needs to be justified in terms of two types of conditions, respectively the necessary and sufficient conditions. While the necessary condition refers to methodological requirements, particularly *when* and *how* a two-sex model should be used, the sufficient condition sets the *whys* for the application of a two-sex methodology itself; it includes the research issue that needs explanation, specific theoretical issues and empirical puzzles, as well as the operational definitions, research hypotheses, and two-sex measures.

Demographers are aware that the complementarity between the sexes works in the daily life of population change. But this awareness is usually drawn from simple commonsense or perhaps the individual experience of researchers. Frequently demographers who investigate demographic change refer to the everyday reproductive role of males and females; but conventional demographic teaching provides no guidelines, nor even discusses when and why demographers should use either neuter, one-sex or two-sex methods. Basic and advanced textbooks and the demographic literature in general continue to shy away from any attempt to explain when and why both sexes should, or should not, be taken into consideration in any scientific study of population.

In the end, even when a two-sex approach would, at least intuitively, seem feasible and appropriate, demography has nonetheless developed as long as demographers have been able to identify the necessary and sufficient conditions to describe and explain specific empirical puzzles. For some purposes a neuter measure (i.e. crude birth rates), equations (i.e. basic demographic equations of population growth) or even theory (i.e. classical demographic transition, Lotka's neuter stable population theory) can provided satisfactory answers; for other purposes one-sex methods and theories (i.e. net reproduction rate, total fertility rate, one-sex stable population theory) are required. In this context, the history of demographic ideas reviewed in Part I and, in particular the revelation in Chapter 6 concerning the evolution of the demographic concept of fertility according to its three bifurcations seems paramount.

These issues are discussed in more detail in Part II and in theoretical terms. However, it has been the historical journey described in Part I that has led to inference that the leitmotif of the evolution of the demographic concept of fertility seems to be the investigations on the feasibility, usefulness, reliability and validity of the neuter, one-sex and two-sex methods in demographic analysis.

The concept of fertility has not developed by definition once and for all; instead, it has grown out of discoveries and by virtue of two main processes: the requirements necessary to apply specific concepts to the analysis of demographic phenomena, and the functions that new operational definitions perform in the explanatory process. Although working concepts such as nuptiality and mating function are important mechanisms in the functioning of demographic reproduction, it is extraordinary that current literature on fertility determinants does not contemplate the feasibility, usefulness, reliability and validity of the two-sex perspective.

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